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THE UNIVERSITY OF ALBERTA
APPROXIMATE ANALYSIS OF SHEAR
WALL-FRAME STRUCTURES

by

SAURENDRANATH GUHAMAJUMDAR



A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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EDMONTON, ALBERTA

MAY, 1968

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled APPROXIMATE ANALYSIS OF SHEAR WALL-FRAME STRUCTURES submitted by SAURENDRANATH GUHAMAJUMDAR in partial fulfilment of the requirements for the degree of Master of Science.

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ABSTRACT

A computer analysis has been developed to predict the behavior of tall buildings subjected to lateral loads. The buildings considered are composed of shear walls and frames. Each shear wall-frame unit undergoes equal lateral deformations and the structure as a whole does not deform torsionally. The analysis is performed on a subassemblage consisting of a lumped shear wall system and a simple substitute frame. The load-deflection relationship for the structure is determined up to the ultimate load. The formation of plastic hinges in the substitute frame has been considered as well as the inelastic action of the wall. In addition, the secondary moments produced by the axial forces in the columns acting through the sway displacements of the structure are considered.

The lumping procedure used to form the subassemblage is discussed and the analysis of the subassemblage in both the elastic and inelastic range, including the $P - \Delta$ effect, is presented. The results of the analysis are compared with those obtained by other methods. The behavior of a shear wall-frame structure is discussed and illustrated by an example.

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Statement of Financial Position

As at 31 December 2019

Assets		
Non-current assets		
Property, plant and equipment	1,200,000	
Intangible assets	300,000	
Financial assets	100,000	
Current assets		
Inventory	200,000	
Trade receivables	400,000	
Trade payables	(100,000)	
Other current assets	100,000	
Current liabilities		
Trade payables	(100,000)	
Other current liabilities	(200,000)	
Equity		
Share capital	1,000,000	
Reserves	200,000	
Provisions	(100,000)	
Other equity	100,000	

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CHAPTER I

INTRODUCTION

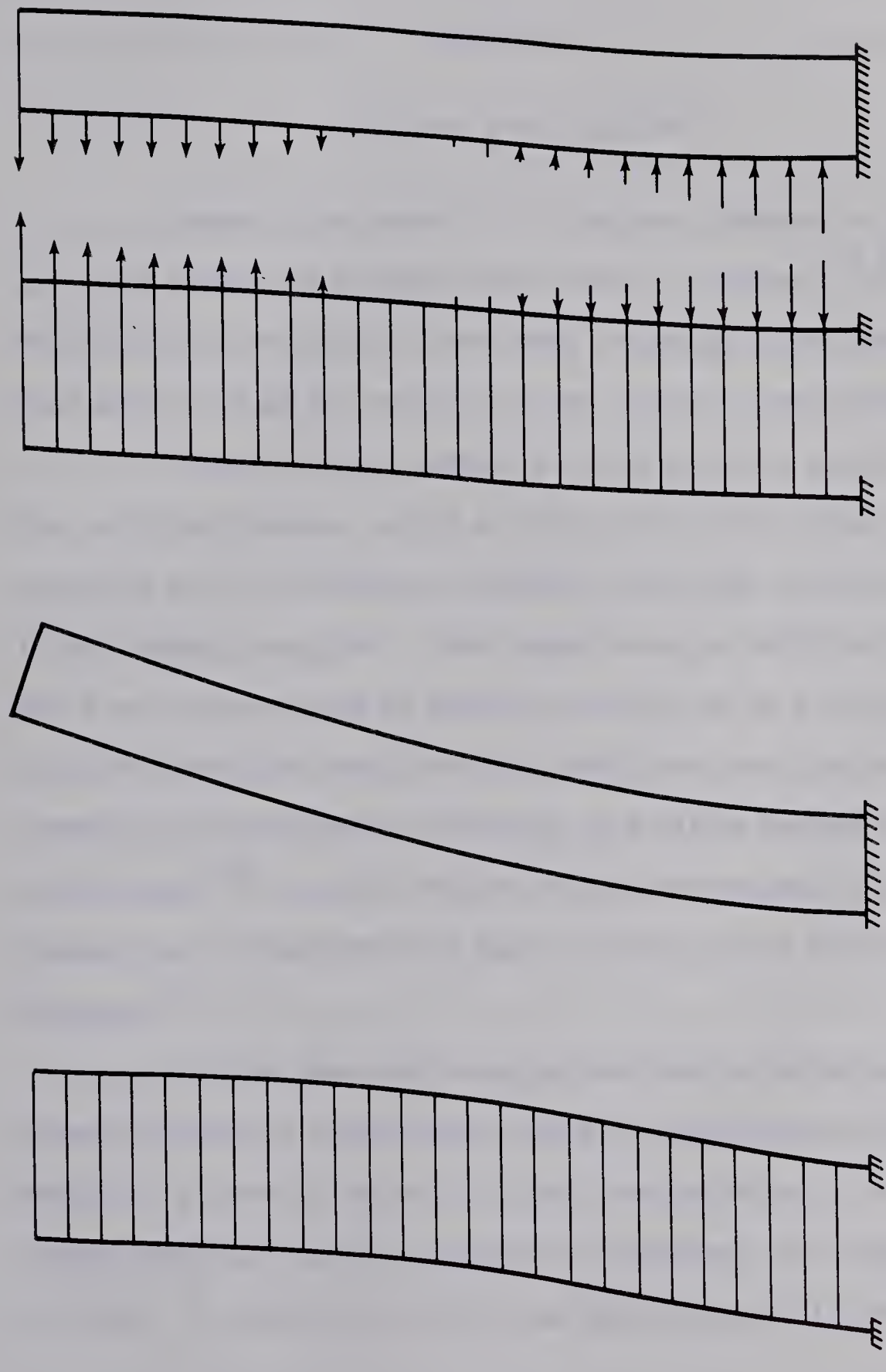
Many types of buildings are composed of shear walls coupled to structural frames. The shear walls may be on the exterior face of the building or in the form of interior cores for elevators or stairs. Shear walls generally have stiffnesses which may be much greater than those of the members composing the structural frame. It has been common to design the frame to resist the vertical loads acting on the structure and the walls to resist the lateral forces ⁽¹⁾. The strength and stiffness of the shear wall was estimated by the same methods used for frame members.

The above procedure is acceptable for low buildings, but as more slender structures are considered, the interaction between the shear wall and the frame becomes significant. If the frame alone resisted the lateral forces it would deflect as shown in FIGURE 1.1a, in this case the sway deformations of each story are approximately equal. On the other hand, the free shear wall would deflect as shown in FIGURE 1.1b. Since the two systems are connected the redistribution of forces will result in a final deflected shape as shown in FIGURE 1.2. The frame system will restrain the wall in the top stories and as a result forces will be induced in the frame that cannot be predicted by assuming the total lateral load to be resisted by the wall ⁽²⁾.

Since the structure under consideration is highly indeterminate, methods which account for the inelastic action will more reliably predict the behavior of the system. In tall buildings, the axial loads in the columns are significant and produce additional overturning moments commonly referred to as $P - \Delta$ moments; where P is the axial load and Δ is the displacement due to sidesway. The effect of these "secondary" moments is to reduce the capacity of the structure to resist lateral loads. These moments must be considered in the analysis of tall building frames ⁽³⁾.

The use of computers has increased considerably during the past decade, and this makes it possible to analyze complicated problems. Computer methods are particularly suitable for elastic-plastic analysis, since the structure must be analyzed in several stages of deterioration, in order to obtain the complete response.

The above discussion clearly indicates the importance and complexity of combined shear wall-frame systems. The present investigation attempts to predict the load-deflection behavior of these systems, both in the elastic and the inelastic range. The $P - \Delta$ moment is included in the analysis and the corresponding reduction in the lateral load-carrying capacity of the structure is examined. Example structures were analyzed and the results are compared to those obtained by other methods.



(a) - FREE FRAME (b) - FREE WALL COMBINED FRAME & WALL

FIGURE 1.1 FREE DEFLECTED SHAPE OF FRAME AND SHEAR WALL

WALL-FRAME SYSTEM

CHAPTER II

PREVIOUS INVESTIGATIONS

Recently the behavior of structures composed of shear walls and rigid frames has aroused a great deal of interest (4,5,6,7,8,9). Methods of elastic analysis have been presented but no real attempt has been made to study the behavior of the system in the inelastic range.

The traditional methods of frame analysis make no provision for wall type elements having a finite width. The girders attached to the walls will be subjected to moments due to the vertical displacement of the connection points. These moments are in addition to those which would be computed by assuming the wall to be a line element. Also, the equations resulting from traditional analyses of structures composed of stiff elements connected to flexible members are ill-conditioned (10) so that the resulting solutions may be inaccurate. However, such structures have been analyzed, and no difficulties reported (11).

Coupled shear walls may be analyzed by replacing the discrete connecting beam by a continuous lamina of equivalent stiffness (12,13,14). However, in order to obtain a closed form solution, it is necessary to assume constant structural properties throughout the height of the building. An iterative solution has been developed for this

problem, which is capable of accounting for changes in configuration of the shear walls ⁽⁴⁾. This procedure is similar to that developed ⁽²⁾ for the solution of combined shear walls and frames and will be discussed in detail below.

Expressions for the lateral stiffness of various types of beam and column arrangements have been developed by Cardan ⁽⁵⁾. Each story is isolated by assuming that the column has an inflection point at mid-height. The moments and forces developed by the various structural arrangements are then assumed to be continuously distributed over the height of the shear wall. A differential equation, expressing the equilibrium of the shear wall is formulated by assuming the structure to have constant properties throughout its height. This last assumption makes the method unsuitable for most practical problems.

Parme ⁽⁶⁾ has combined the resistance of the shear wall with that of the frame to obtain an expression for the total lateral load on a floor as a function of the stiffness of members (including the shear wall) and the displacements and rotations of the floor considered and the adjacent floors. The response of the shear wall is formulated in terms of finite difference equations. Finally, the solution of the resulting set of simultaneous equations determines the lateral displacement of each floor. The analysis neglects the axial deformation of the columns and the effect of the wall width on the moments in the girders adjacent to the wall.

FIGURE 2.1 shows the model used by Gould ⁽⁷⁾. Each story is represented by rigid bars connected by extensional and rotational springs. The individual story properties and the effect of adjacent floors are considered in determining the lateral resistance of a particular story. The analysis assumes constant story heights and constant shear wall stiffness and neglects the effect of axial deformations of the columns and shear deformations of the members. The lateral deformations of the wall are represented by finite difference expressions. This procedure leads to a set of simultaneous equations, one for each story.

As described above, the matrix methods have also been used by Clough, King and Wilson ⁽¹¹⁾ to analyze frame shear wall structures. These procedures include the effects of wall width and axial distortions. No attempt was made to perform an inelastic analysis or include the secondary moments due to the $P - \Delta$ effect.

Bandel ⁽⁸⁾ expressed the deformed shape of the combined truss (or wall) and frame system by using a power series. Expressions for the total internal and external work of the system are derived and the constants of the actual deformation functions are determined from the solution of a set of simultaneous equations. The equations result from the minimization of the total potential energy of the system. It is necessary in this case to replace the shear wall by an equivalent truss system.

Rosenbleuth and Holtz ⁽⁹⁾ assumed that an initial configuration may be obtained from the differential equation solution, by assuming the structure to be completely uniform. This initial configuration is then improved in later cycles of iteration by considering the actual properties of the structure.

Khan and Sbarounis ⁽²⁾ have developed an iterative method, in which the building is lumped to form a structural model (FIGURE 2.2). The model consists of a shear wall and a simplified frame system. The elastic analysis of the present work is based on a similar approach. The method accounts for the variation in wall and frame properties. The effects of base rotation, plastic rotation of the wall and secondary deflections in the frame have also been discussed. Influence curves are provided to estimate the distribution of shear between the wall and the frame for a wide range of structural proportions.

The above discussion summarizes the existing literature dealing with the analysis of shear wall-rigid frame structures in the elastic range. So far no attempt has been made to determine the inelastic behavior of such structures. Literature is available, however, concerning the inelastic behavior of unbraced multi-story planar frameworks acting without shear walls.

Several theories are available to predict the behavior of multi-story frames. In order to correlate the results of the analysis with test results, however, certain factors must be accounted for. As the load is increased on the frame, portions of the member will yield, thus decreasing the stiffness of the frame under additional loading. For tall slender structures the secondary moments, produced by the axial column loads acting through the sway displacements, reduce the capacity of the frame to resist lateral loads. In addition, for these structures,

the redistribution of moment caused by column shortening may become significant. It has been found that an elastic-plastic analysis which includes the above effects will closely predict the response of the structure up to the ultimate load.

The second-order elastic-plastic analysis of the complete structure can be performed using the displacement method (15,16,17). A second-order elastic analysis is used up to the formation of the first hinge. The stiffness is then reduced by inserting an idealized hinge in the structure at this point. The analysis is then continued for successive increments of load. At each load increment, this procedure results in a set of simultaneous equations which are the equilibrium equations for the deformed structure. The joint rotations, vertical deflections, and story translations are determined from the solution of the above set of equations. The end forces in the members are then determined by substitution. Each time a new hinge is formed, the determinant of the stiffness matrix, before and after hinge formation, is calculated. When the determinant passes from a positive value to zero or a negative value, the maximum capacity of the structure has been attained.

Isolated parts of a multi-story frame have been analyzed as independent units (3). The points of inflection in the columns are assumed to be at mid-height, so that two imaginary horizontal slices through the structure isolate one floor level with its attached columns as shown in FIGURE 2.3. Subsequently this model is reduced to four

subassemblages, each consisting of a column restrained by the adjoining beams. The load-deflection curve for each subassemblage is obtained by performing an inelastic analysis; the load-deflection relationship for a particular story is then obtained by summing the responses of the individual subassemblages.

In the case of frame-shear wall structures, the behavior of a portion of the shear wall cannot be isolated. The deflections and rotations of each part of the wall depend on the deformation of the wall as a whole. Thus, a subassemblage, which would include the influence of the shear wall, cannot be obtained in a rational manner.



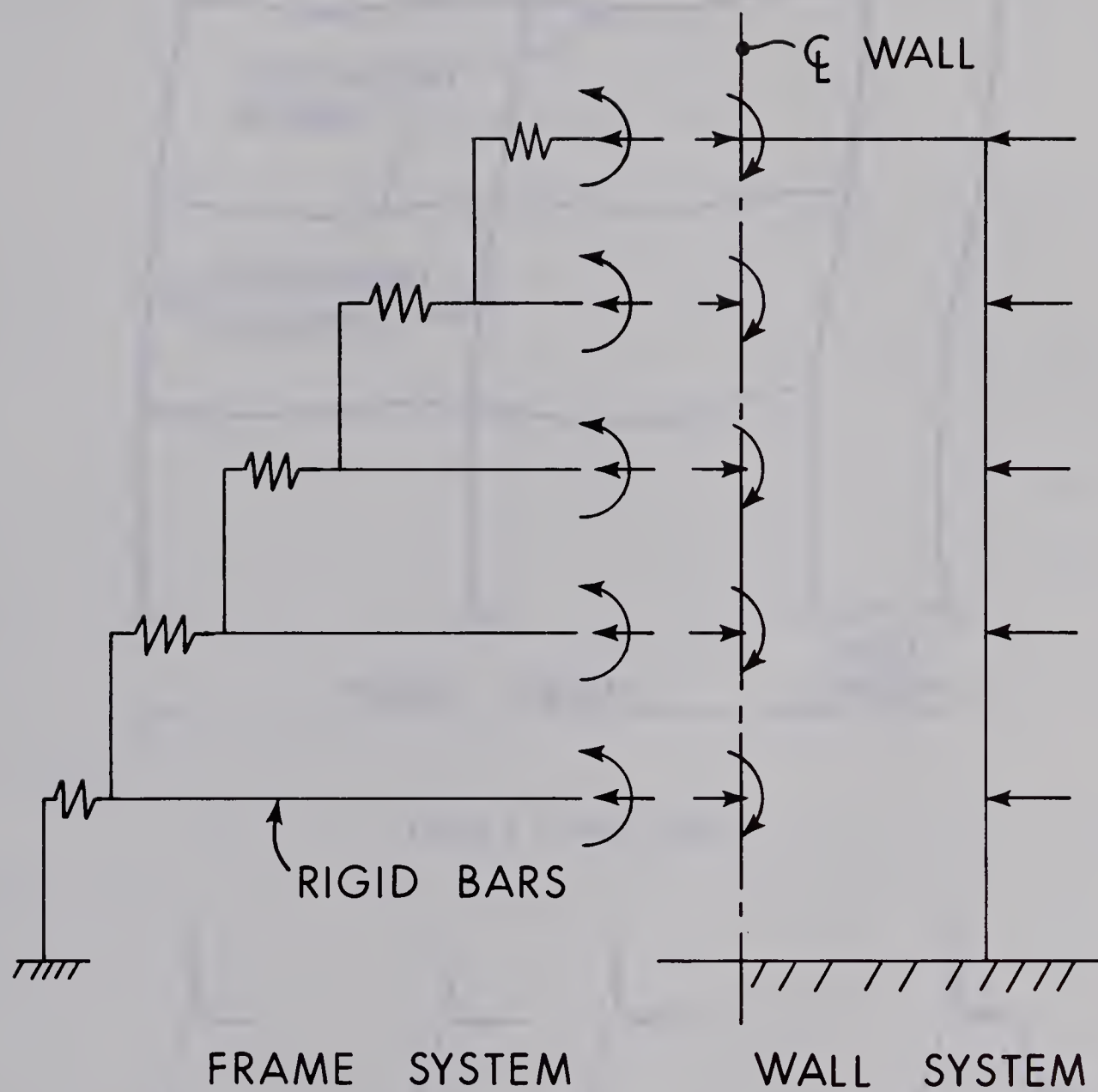


FIGURE 2.1 GOULD'S MODEL

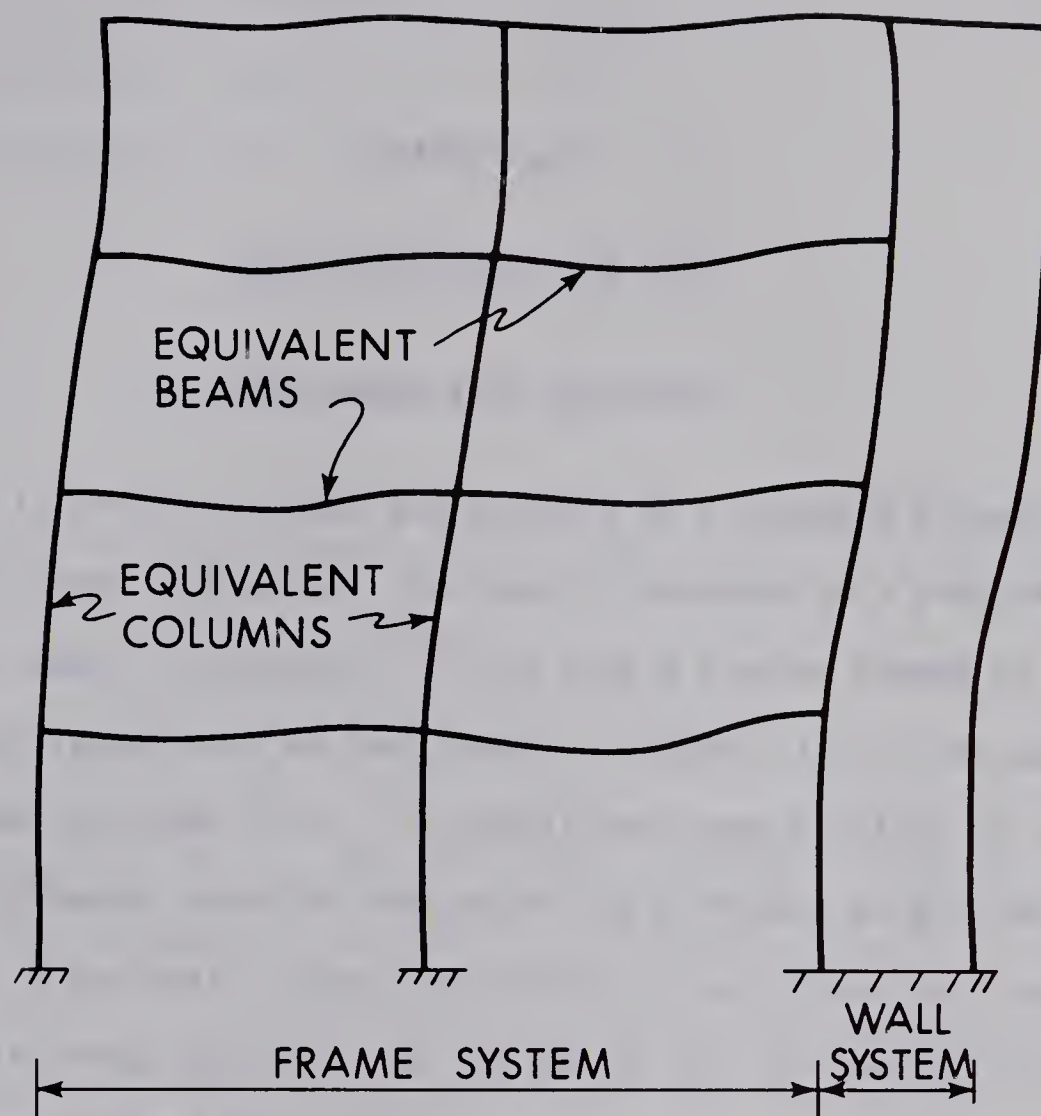


FIGURE 2.2 KHAN'S MODEL

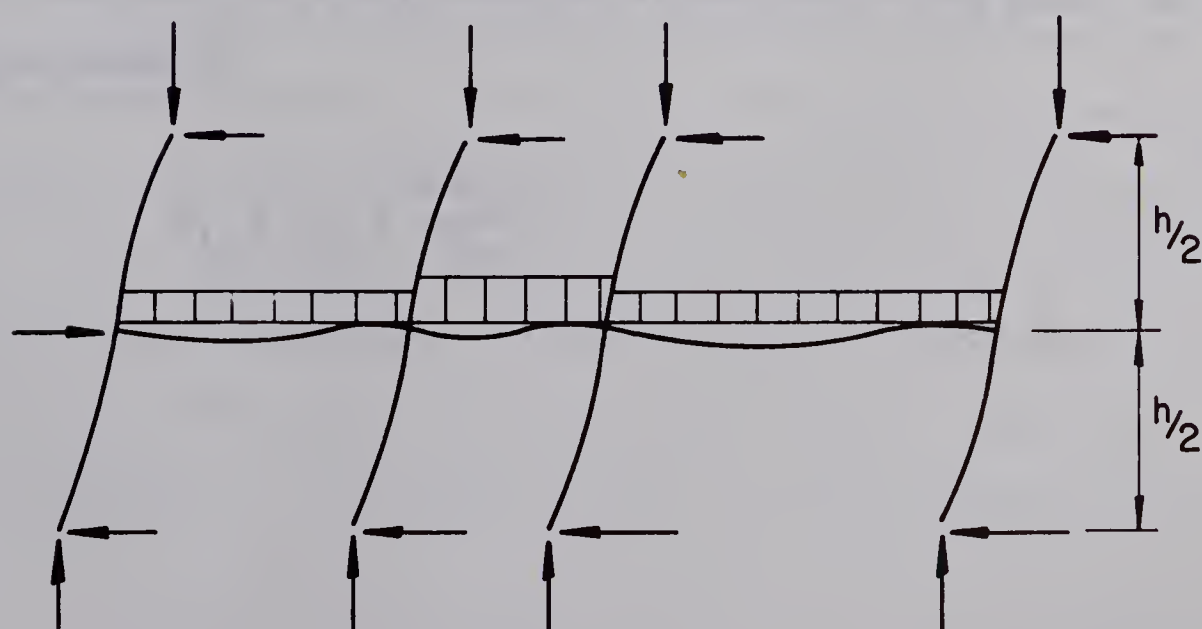


FIGURE 2.3 CENTRAL PORTION OF TWO STORY UNIT

CHAPTER III

ANALYTICAL MODEL FOR THE

FRAME-SHEAR WALL STRUCTURE

In order to reduce the analysis of a complete structure to manageable terms, the actual structure is replaced by a simplified structural model. Lightfoot ⁽¹⁸⁾ has used a similar scheme to replace a multi-story frame, such as that shown in FIGURE 3.1a, by the equivalent frame shown in FIGURE 3.1b. The substitute frame consists of columns having stiffnesses equal to the sum of the stiffness of all the actual columns in a particular story, restrained at each floor by a beam, having a stiffness equivalent to the sum of the stiffnesses of all the actual beams at that floor level.

In FIGURE 3.1a and 3.1b, K_{12} , K_{21} , K_{23} etc. are the beam end moments developed for a unit rotation at each end of the beam. For prismatic members,

$$K_{12} = K_{21} = \frac{6 EI_{12}}{L_{12}}$$

In the above expression I and L represent the moment of inertia and length respectively of beam 12.

The equivalent beam restraint, K_{BE} , at each floor is given by

$$K_{BE} = 2 (K_{12} + K_{23} + K_{34})$$

and the stiffness of the equivalent column, K_{CE} , is given by

$$K_{CE} = K_1 + K_2 + K_3 + K_4$$

where K_1 , K_2 , K_3 and K_4 are the column stiffnesses in the story.

The equivalent frame can then be analyzed under the given external loads. The moments in the actual frame can be determined from the analysis of the substitute frame, since the moments and forces in each member are proportional to the ratio of the stiffness of the actual member to the stiffness of the equivalent member.

Frischmann and Prabhu ⁽¹⁹⁾ have adopted a similar method to lump the member stiffnesses of multi-story framed structures and have also used the method to analyze inter-connected shear wall structures.

In the present analysis, the actual shear wall-rigid frame structure has been lumped into a structural model consisting of two systems: the Frame System and the Shear Wall System. These are shown in FIGURE 3.2. In FIGURE 3.2, $B_i F_i$ is the equivalent beam

representing the actual beams which span between two columns;
 B_i W_i is the equivalent beam representing the actual beams which link the columns to the shear wall at the i th floor of the structure. In the model the shear walls in the actual structure have been replaced by a single shear wall with an equivalent stiffness and width. Similarly the columns in the actual structure have been lumped to form an equivalent column. In the analysis of the model, joints B_i and F_i are subjected to equal rotations and F_i is free to translate laterally. The rotational base springs for the column and the shear wall reflect the foundation conditions.

Building frames are made up of various combinations of frames and shear walls. FIGURE 3.3 shows the plan of a building consisting of two exterior frames and a central shear wall. The lateral loads are assumed to be applied in a direction parallel to the exterior frames. Assuming the floors to be infinitely rigid in their own plane, so that all points on a floor are forced to undergo equal lateral translations, the building in FIGURE 3.3 can be replaced by the system shown in FIGURE 3.4. It has been assumed that the floor slabs offer no rotational restraint to the shear wall, so that the system resisting the lateral loads consists only of the two exterior frames linked to the shear wall. In FIGURE 3.4, K_C and K_B are the stiffness (EI/L) of each individual column and beam in the building.

The system in FIGURE 3.4 can be replaced by the equivalent system shown in FIGURE 3.5. In any story, the columns are lumped into a single equivalent column. The frame system consisting of the equivalent column restrained by a beam is linked to the wall system. In FIGURE 3.5, it has been assumed that both ends of the left beam will undergo equal rotations. Since the restraint offered by the beam must be equivalent to the restraints on the column shown in FIGURE 3.4, the stiffness of the left beam in FIGURE 3.5 is given a stiffness equal to twice the sum of the stiffnesses of all the beams in the actual structure.

In other types of buildings, beams may directly connect adjacent columns to the shear wall system. Generally these beams offer relatively little restraint to the lateral deflection of the structure and a simple and conservative way of forming the analytical model would be to neglect the action of these members and lump the remaining beams in the left beam as before. This would neglect the restraint offered by the beams or slabs connected directly to the shear wall. If this restraint is significant, the problem of finding an equivalent system for such a building becomes complex and a complete solution has not been attempted.

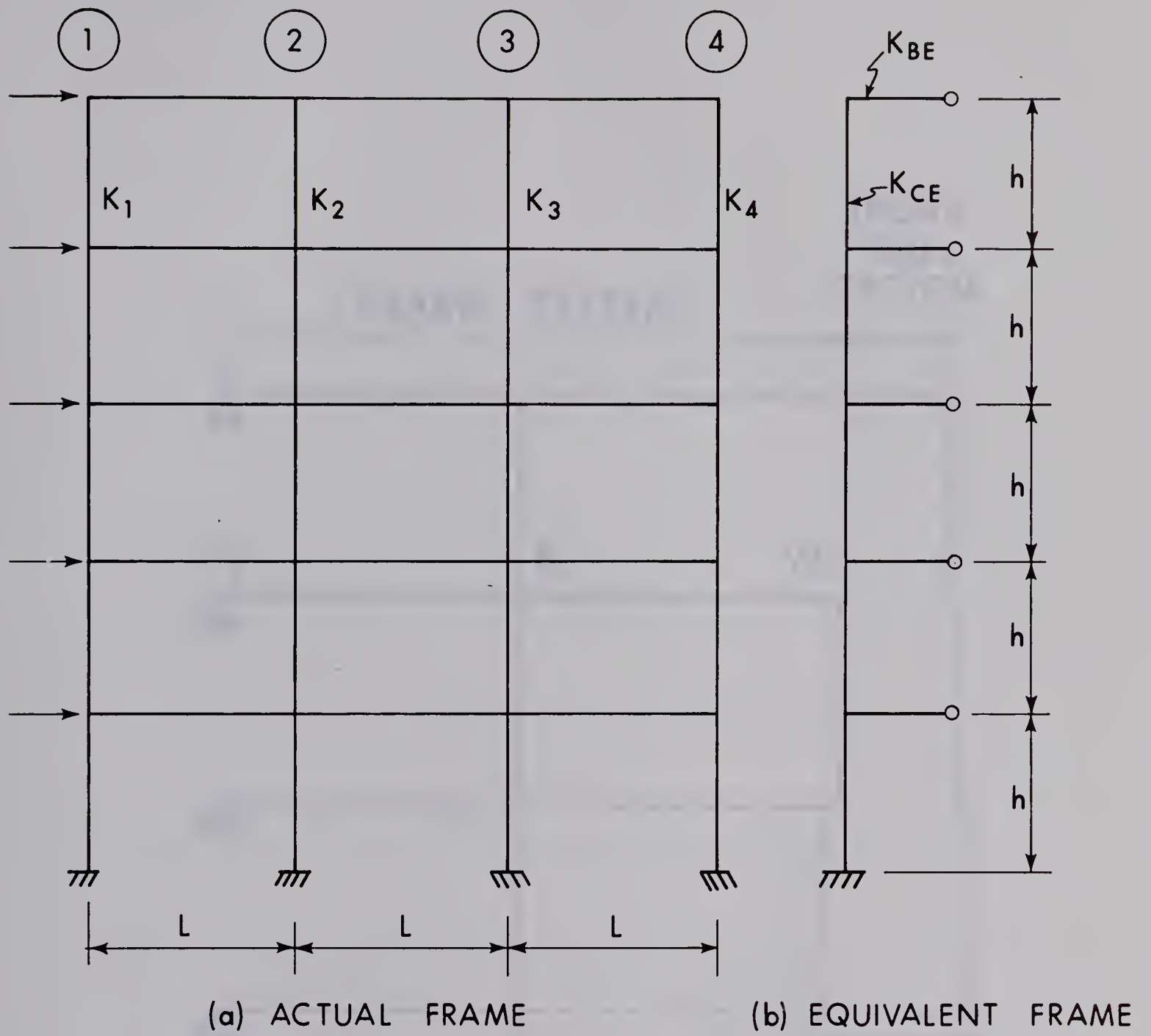


FIGURE 3.1 LIGHTFOOT'S SUBSTITUTE FRAME

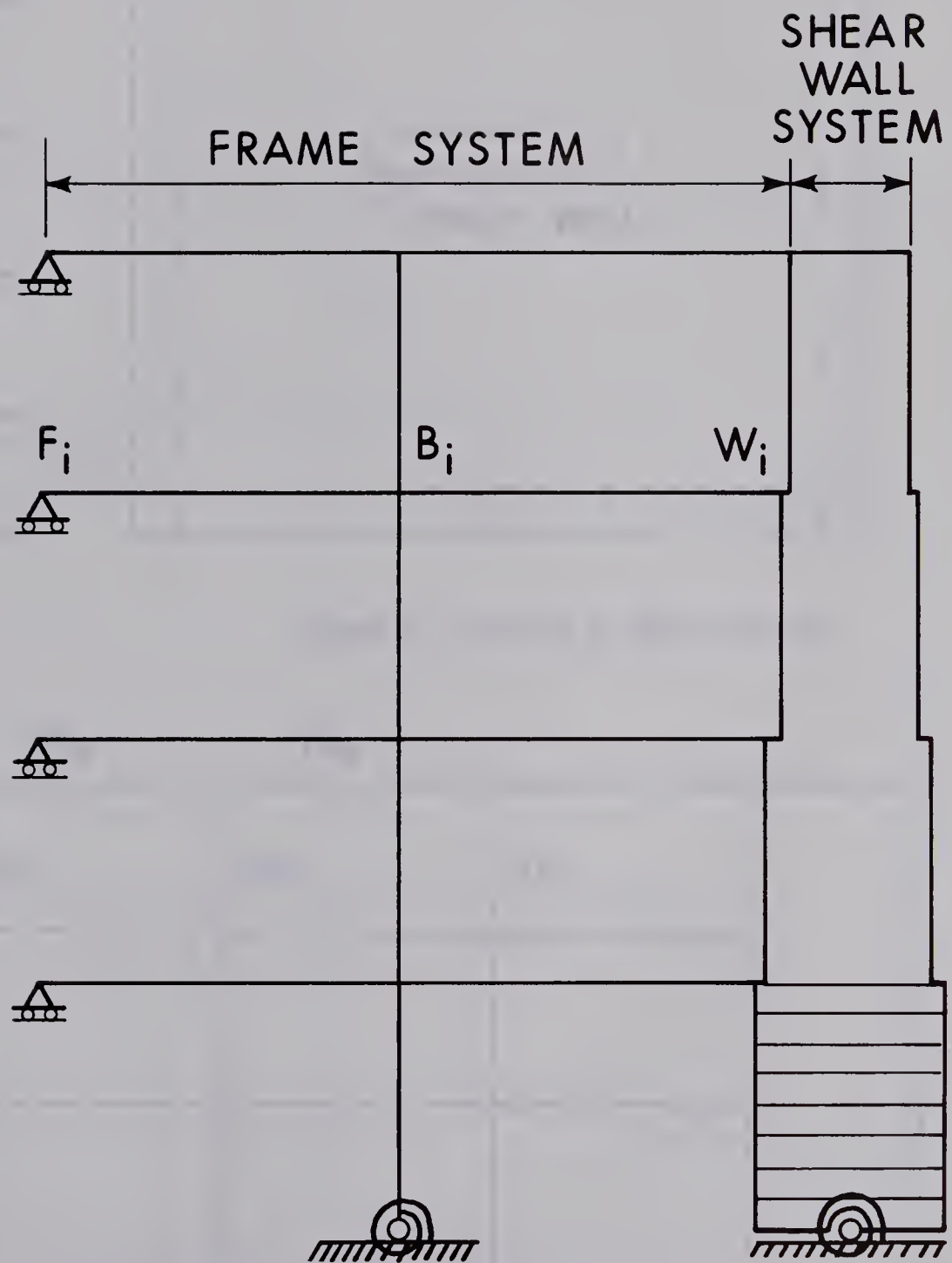


FIGURE 3.2 ANALYTICAL MODEL

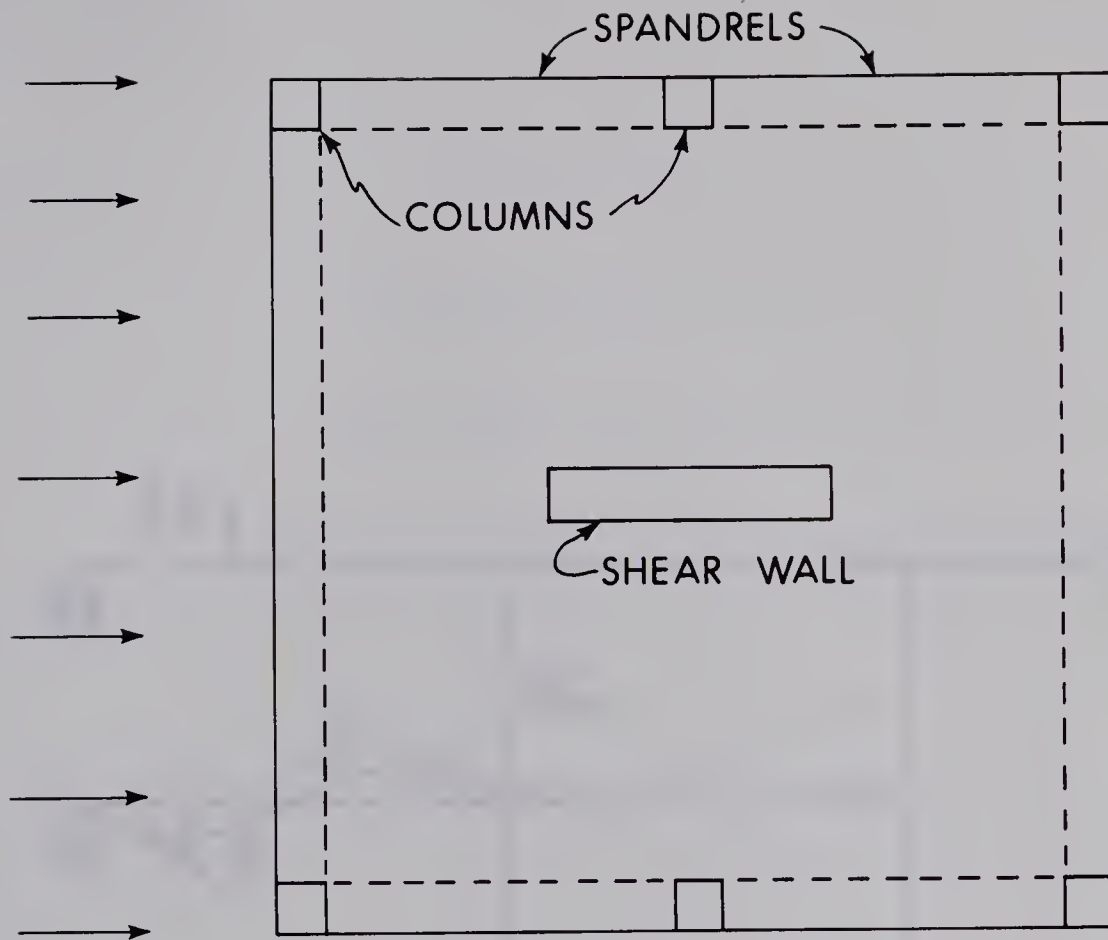


FIGURE 3.3 PLAN VIEW OF EXAMPLE BUILDING

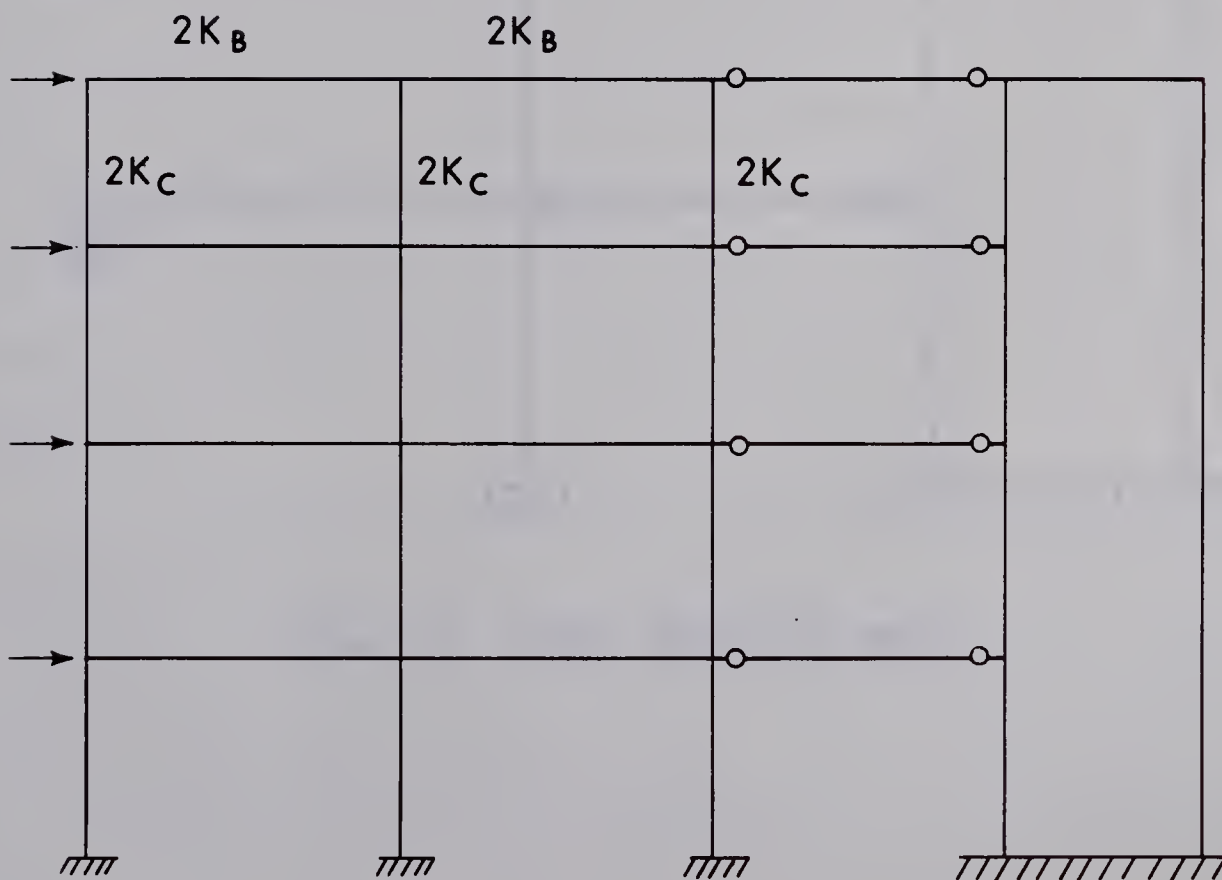


FIGURE 3.4 LUMPED EXTERIOR FRAMES

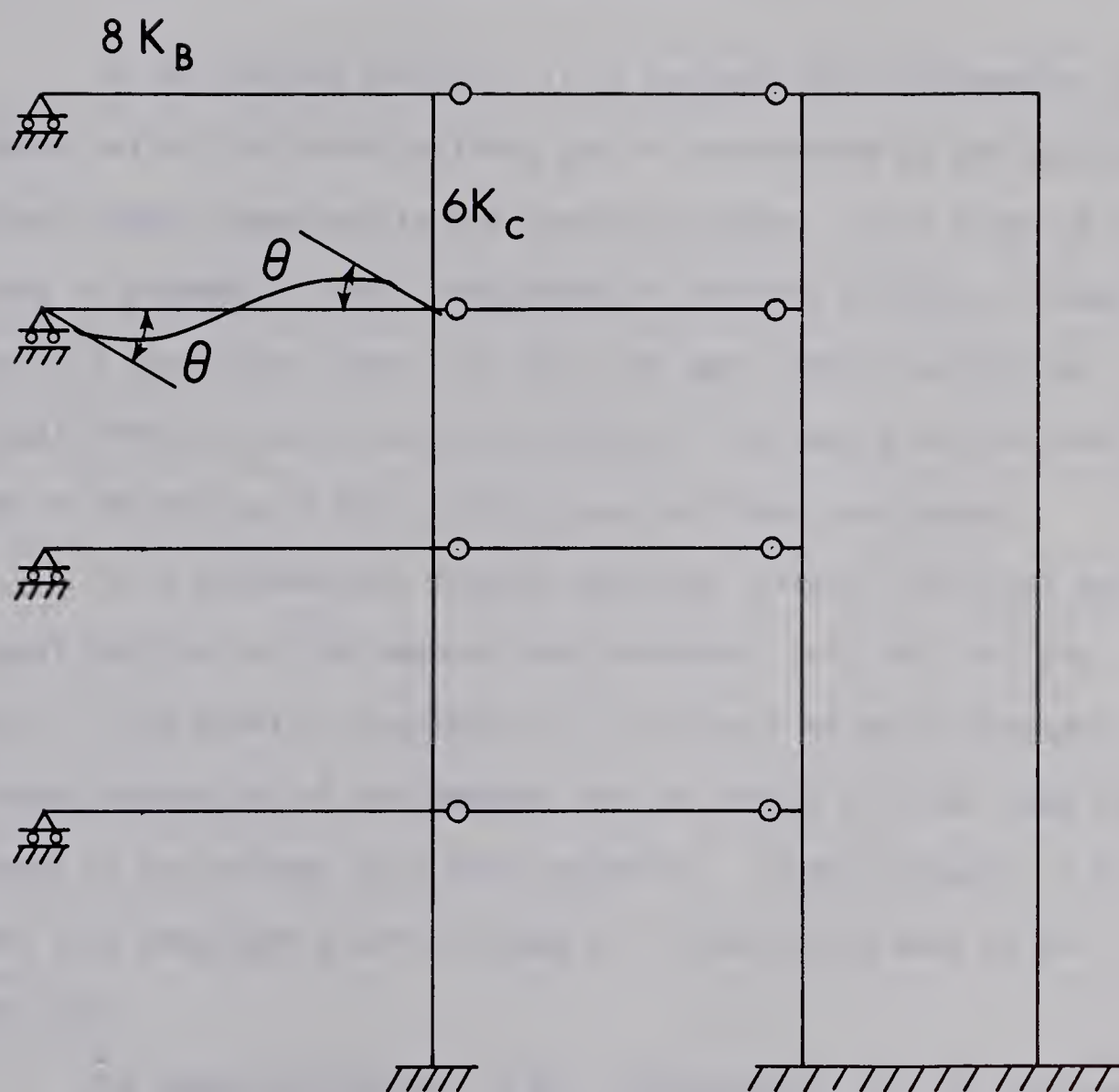


FIGURE 3.5 LUMPED ANALYTICAL MODEL

CHAPTER IV

ANALYSIS OF MULTI-STORY

FRAME-SHEAR WALL STRUCTURES

In the present analysis, it is assumed that the behavior of the shear wall-rigid frame building can be represented by the substitute structural model described in the previous chapter. Each floor of the building is assumed to form a diaphragm of infinite rigidity. Thus all points on a particular floor will have the same lateral deflection. Torsional effects, due to non-coincidence of the centre of load and the centre of resistance of the building have not been considered.

It is assumed that lateral buckling, lateral-torsional buckling and local buckling of the members are prevented; only the in-plane behavior of the model is considered. The effects of axial shortening and shear deformation of the members and the effect of axial load on the stiffness of the columns have been neglected. In the columns, it is assumed that idealized plastic hinges will form at the ends of the members only.

The moment-curvature ($M-\theta$) relationships for the individual members of the frame are assumed to be ideally elastic-plastic as shown in FIGURE 4.1. In FIGURE 4.1, M_{pc} represents the plastic moment capacity of the member, reduced for axial load (in the columns) while θ_{pc} represents the curvature corresponding to M_{pc} , assuming ideally

elastic behavior.

The moment-curvature relationship for the shear wall system is read into the computer program as a series of moment and curvature co-ordinates. FIGURE 4.2 shows a typical plot of the moment, M , non-dimensionalized as M/M_p , versus the curvature, ϕ , non-dimensionalized as ϕ/ϕ_p ; where M_p is the plastic moment capacity of the wall and ϕ_p is the elastic curvature corresponding to M_p ($\phi_p = M_p/EI$). For the examples that follow only two branches of the $M-\phi$ curve have been used.

The loading is assumed to be applied statically. The structure is analyzed under a constant vertical load and monotonically increasing lateral load. The analysis begins by applying the entire lateral load to the shear wall. The corresponding shape of the deflected shear wall is shown by the dashed line in FIGURE 4.3. The deflection at the i th level is $\Delta_{w i}^{(1)}$ where i represents the floor under consideration and the superscript (1) represents the initial iteration cycle. The frame is forced into this deflected shape and the corresponding (negative) frame shears are computed. The shear wall is reanalyzed by applying the negative shears from the frame analysis. The resulting deflected shape is shown by the solid line in FIGURE 4.3. The net deflection from the previous position of the shear wall is $\Delta_{w i}^{(2)}$. The frame is again forced into the corresponding deflected shape and the negative shears recomputed. In the actual process used, the frame is not forced into the deflected shape characterized by $\Delta_{w i}^{(2)}$ but instead a forcing formula is applied to speed convergence. The new position is shown by the broken line and is characterized by the deflection $\Delta_{w i}^{(3)}$. The

above process, and the necessary modifications, are described in detail in the following paragraphs.

A computer program has been developed to assist in the analysis of shear wall-frame structures. The program takes into account yielding in the frame, the inelastic action of the shear wall and the secondary moments caused by the $P-\Delta$ effect. The axial force, P , on the structure, acting through a sidesway displacement, Δ , causes an overturning moment in addition to that produced by the lateral loads, known as the $P-\Delta$ moment. To account for this effect, the equilibrium equations are formulated on the deformed structure. This "secondary" moment causes significant reductions in frame strength and must be included in the analysis of tall, slender structures ⁽³⁾.

The method used to analyze the model consists of a series of iterations. The steps involved are described below:

- (A) The vertical load acting on the frame is computed. This load remains constant while the lateral load on the structure is incremented to determine the load-deflection relationship. The initial value of the lateral load is selected so that the structure remains elastic.
- (B) Each story of the shear wall is divided into segments. The entire lateral load is applied to the shear wall and the moment at the centre of each segment is calculated by considering the wall as a free cantilever. The rotations and deflections are calculated at every segment using moment-area principles. The vertical displacement at the junction

of the shear wall and the frame is computed as the product of the rotation of the wall and one-half the wall width. FIGURE 4.4 shows the deflected shape of the shear wall under the entire lateral load, where $\theta_{W i}^{(1)}$, $\Delta_{W i}^{(1)}$ and $\delta_{W i}^{(1)}$ are the rotation, lateral displacement and vertical displacement of the wall. The initial subscript W represents the shear wall, while i represents the floor under consideration and the superscript (1) represents the initial iteration cycle. The story height is $H_{S i}$.

- (C) The frame system is forced into the deflected shape of the wall as defined by the deformations computed above. This step is shown in FIGURE 4.5.

Next the frame joint rotations are computed. In FIGURE 4.5, B_i is the beam-to-column joint, W_i is the joint at the connection of the wall and the right side beam and F_i is the left end of the left side beam. Under lateral loads beam $B_i F_i$ will have a point of contraflexure approximately at midspan. This is enforced in the structural model by assuming that the joints B_i and F_i have equal rotations and that the left end support, F_i , is free to translate.

At each joint the slope-deflection equations are written and by using the moment equilibrium equation, an expression is developed which relates the joint rotation to the deformations of the adjacent joints and the member properties. The detailed derivation of this equation is given in APPENDIX A.

A set of equations is generated by applying the moment equilibrium equation to all the beam-to-column joints in turn. The joint rotations are then computed using the Gauss-Seidel Iteration Method. The moments at potential hinge locations in the members of the substitute frame can now be computed. Next the vertical and horizontal shears at the junctions of the wall and frame system are computed.

- (D) At each floor level, the moment to be applied to the wall system for the next cycle of iteration, $M_{W i}$, is the algebraic sum of the moment, $M_{WB i}$, at the end W_i of the right beam and the moment produced by the beam shear multiplied by half the wall width. FIGURE 4.6 shows typical forces at floor level i . In the initial iteration, the moment, $M_{W i}$, is assumed to be zero. This is corrected in subsequent cycles.
- (E) The net out-of-balance forces are applied to the wall and the resulting deflections, $\Delta_{W i}^{(2)}$, rotations $\theta_{W i}^{(2)}$ and vertical displacements $\delta_{W i}^{(2)}$ are computed as described in Step (B).
- (F) In order to speed convergence, the frame is not forced into the deformation mode defined by $\Delta_{W i}^{(2)}$, etc. Instead the frame is forced into the position defined by:

$$\theta'_{W i}{}^{(2)} = \frac{\theta_{W i}^{(1)} \cdot \theta_{W i}^{(1)}}{\theta_{W i}^{(1)} - \theta_{W i}^{(2)}} \dots \dots \dots (4-1)$$

$$\Delta'_{W i} (2) = \frac{\Delta_{W i} (1) \cdot \Delta_{W i} (1)}{\Delta_{W i} (1) - \Delta_{W i} (2)} \dots \dots \dots (4-2)$$

$$\text{and } \delta'_{W i} (2) = \theta'_{W i} (2) \cdot D_{W i} / 2 \dots \dots \dots (4-3)$$

where $D_{W i}$ is the width of the shear wall at the i th floor. The analysis now returns to Step (C) and the steps (C) to (F) are repeated. For any cycle (after the initial cycle), the forcing Equations (4-1), (4-2), and (4-3) are replaced by:

$$\theta'_{W i} (n) = \frac{\theta_{W i} (1) \cdot \theta_{W i} (n-1)}{\theta_{W i} (n-1) - \theta_{W i} (n)} \dots \dots \dots (4-4)$$

$$\Delta'_{W i} (n) = \frac{\Delta_{W i} (1) \cdot \Delta_{W i} (n-1)}{\Delta_{W i} (n-1) - \Delta_{W i} (n)} \dots \dots \dots (4-5)$$

$$\text{and } \delta'_{W i} (n) = \theta'_{W i} (n) \cdot D_{W i} / 2 \dots \dots \dots (4-6)$$

where n denotes the cycle in progress.

For any cycle (except the initial cycle) convergence tests for rotations and deflections are performed at the end of Step (D) to determine whether further iteration is required.

The system has converged if the quantities $\frac{\theta'_{W i} (n) - \theta'_{W i} (n-1)}{\theta'_{W i} (n)}$

and $\frac{\Delta'_{W i}^{(n)} - \Delta'_{W i}^{(n-1)}}{\Delta'_{W i}}$ are less than a specified limit

(normally 0.01).

- (G) If the system has not converged as defined above, then force F_i and moment $M_{WB i}$ (FIGURE 4.6) at each floor are applied on the shear wall and the Steps from (E) to (G) are repeated.
- (H) On the other hand, if the system has converged, the P- Δ effect is included. The total axial load at each floor level of the structure has been read into the program. The increased story shears due to the P- Δ effect are simulated by increasing the applied lateral loads. The additional force $H'_1 i$ is calculated on the basis of the deformations obtained from the analysis under the lateral loads H_i . The derivation is given in APPENDIX B. Next the lateral loads $(H_i + H'_1 i)$ are applied on the shear wall system and Steps (B) to (G) are repeated. The deformations of the structure obtained from this analysis are compared with the deformations obtained from the analysis under lateral loads H_i only.
- (I) If the deformations obtained from the two analyses do not agree (within a specified limit) new additional forces $H'_2 i$ are computed as above using the deformations of the structure taken from Step (H). Steps (B) to (I) are then repeated.
- (J) If the deformations agree the shear resisted by the frame is then deducted from the total lateral load to determine the shear to be resisted by the wall.

The formation of hinges in the structure will change the moment equilibrium equations at each beam-to-column joint. FIGURE 4.7 shows the possible hinge locations between stories $i-1$ and $i+1$. The modified moment equilibrium equations are developed in APPENDIX A.

At the end of each load increment the deflected shape of the structure is obtained as well as the distribution of forces and moments. In order to determine the load-deflection curve for the structure, the lateral load is incremented and the Steps (B) to (L) are repeated.

For each increment of lateral load, the structure is treated as a separate problem. The manner of including the inelastic action of the structure implies that the stiffness deterioration in the analysis lags, by one load increment, the actual stiffness deterioration. To minimize this effect, large load increments are used in the elastic range and smaller increments in the inelastic range of behavior.

The analysis was performed using a Fortran Program written for the I.B.M. 360 Computer. By slightly modifying the basic program it is possible to use the program for elastic first order, elastic second order or inelastic analysis. The printout of the Fortran Program has been included as APPENDIX C.

In the 'MAIN' program, the number of problems to be solved is read in. The flow chart is shown in FIGURE 4.8. Then Subroutine 'SR' is called. The flow chart for Subroutine 'SR' is shown in FIGURE 4.9.

This subroutine reads in the data required for the analysis of the first structure. It computes the co-ordinates of each floor from the base and the stiffness of all the members. All the input quantities are printed out. Control is then returned to the 'MAIN' program.

The maximum number of load increments to be used in the analysis is specified. The initial load system (i.e. first increment) is applied to the shear wall system.

Subroutine 'BAKA' is called. The flow chart is given in FIGURE 4.10. 'BAKA' computes the distribution of lateral load between the frame and the shear wall systems. The moments and deformations at each segment of the wall are computed. The convergence formula is then applied (except in the first cycle) to the deflections and rotations. The deformations as computed by the convergence formula are enforced on the frame system. The joint rotations of the frame system are computed in Subroutine 'FRAME' (FIGURE 4.11) by the Gauss-Seidel Iteration Method. In 'BAKA', the moments at all locations are compared with the plastic moment capacities (except during the first load increments). If a hinge is detected, the moment is set equal to the plastic moment capacity for all subsequent load increments. The deformations are compared with those of the previous cycle of iteration. If the deformations are within 1% of one another, control returns to the 'MAIN' program. If the deformations are not within the 1% limit, the process is repeated within 'BAKA'. In this manner the forces and deformations (elastic) for a given loading condition are completely known.

Equivalent horizontal forces simulating the $P-\Delta$ effect are computed. These are added to the initial horizontal load and the total

forces are then applied to the structure. The structure is analyzed by recalling Subroutine 'BAKA'. The deflections obtained from the two analyses (i.e. with and without axial load) are compared; if they are within 1% , the results of the last analysis are printed out by Subroutine 'ROFA' (FIGURE 4.12) and 'SR 3' (FIGURE 4.13).

On the other hand, if the deflections are not within 1% , new equivalent horizontal loads, are computed on the basis of the latest set of deflections. The process is repeated until convergence is achieved.

Inelastic action of the shear wall is considered in the 'MAIN' program. At each segment of the shear wall, the computed moment is compared with the plastic moment capacity. If the moment exceeds the plastic moment capacity of the story, the moment of inertia of the plastified wall segment is reduced according to the $M-\phi$ diagram. This reduced stiffness is used in subsequent steps in the analysis.

In Subroutine 'SR 1' (FIGURE 4.14), the formation of plastic hinges in the members of the frame system is detected. If a hinge is detected, the location of the plastic hinge is printed out with the magnitude of the moment. Control then returns to 'MAIN' program.

If the structure is still elastic, the horizontal load is incremented and the structure is reanalyzed. On the other hand, if a hinge has formed at some location in the structure, the horizontal load is decremented so that the structure is still elastic. The structure is then reanalyzed with this reduced load. Subsequent increments in load are reduced in magnitude to trace the inelastic range as closely as possible. The program stops when the deformations of the structure are unusually large. It then returns to beginning to analyze the next structure.

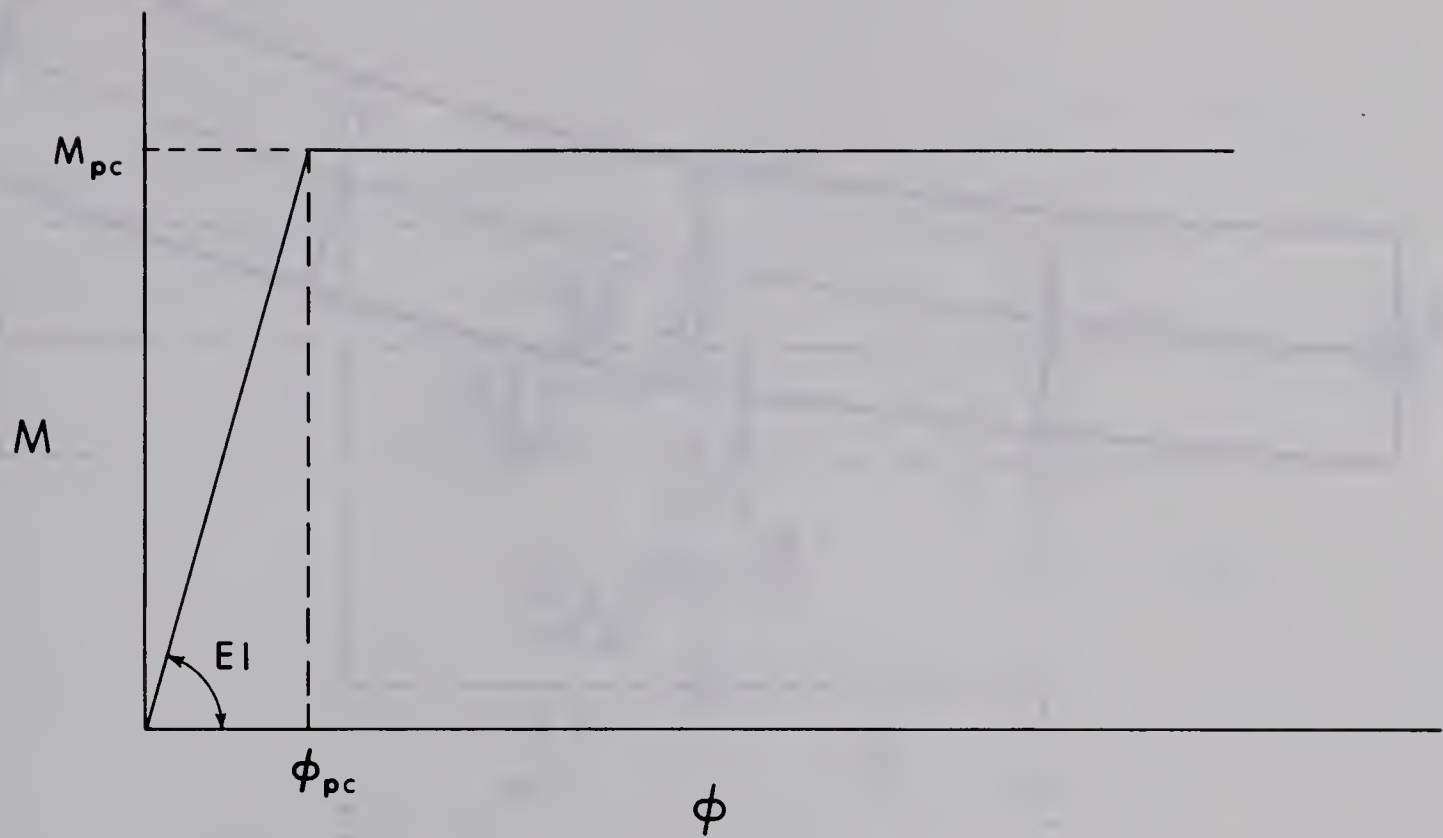


FIGURE 4.1 IDEALIZED MOMENT-CURVATURE DIAGRAM - FRAME MEMBERS

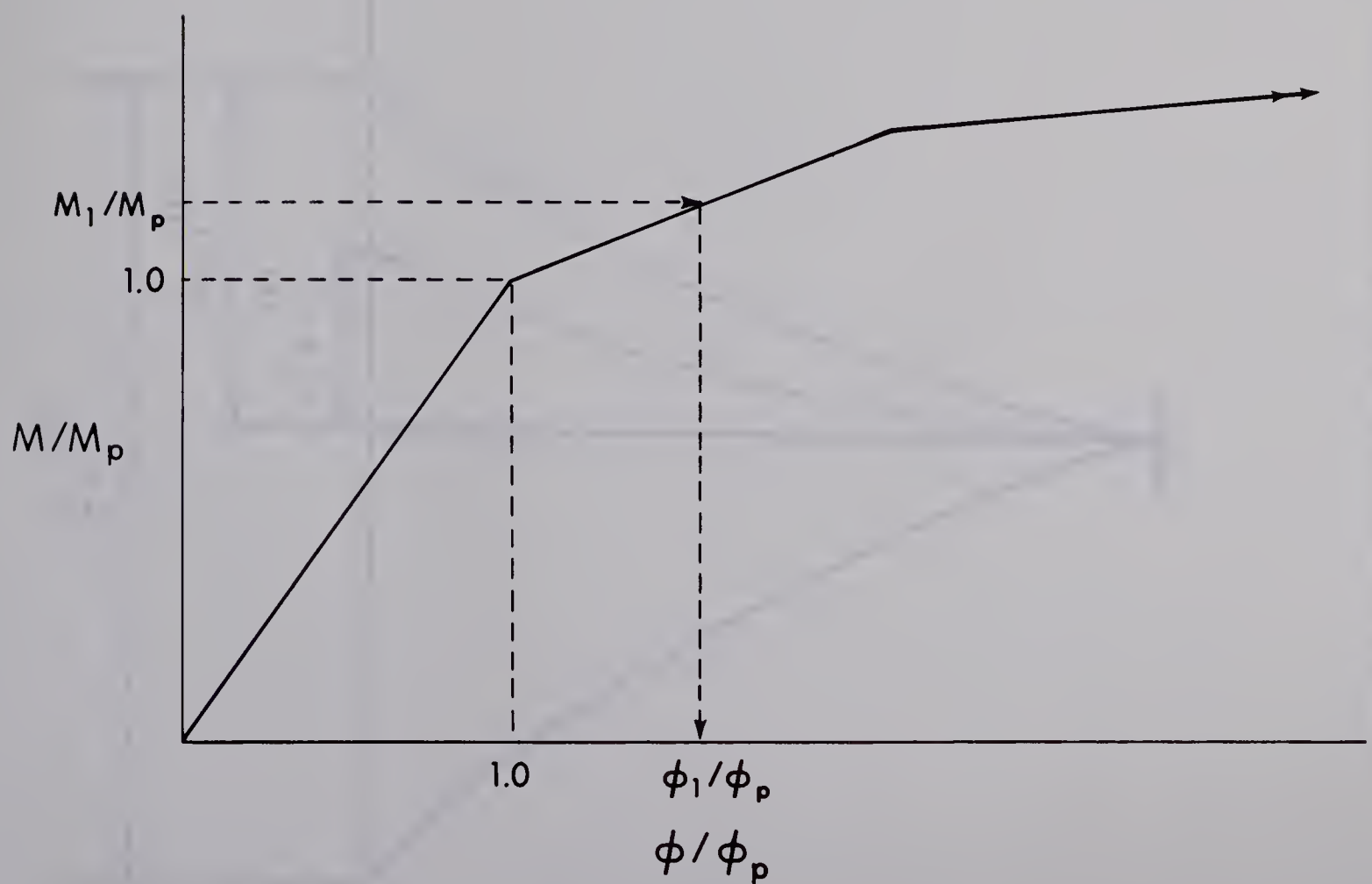


FIGURE 4.2 IDEALIZED MOMENT-CURVATURE DIAGRAM - SHEAR WALL

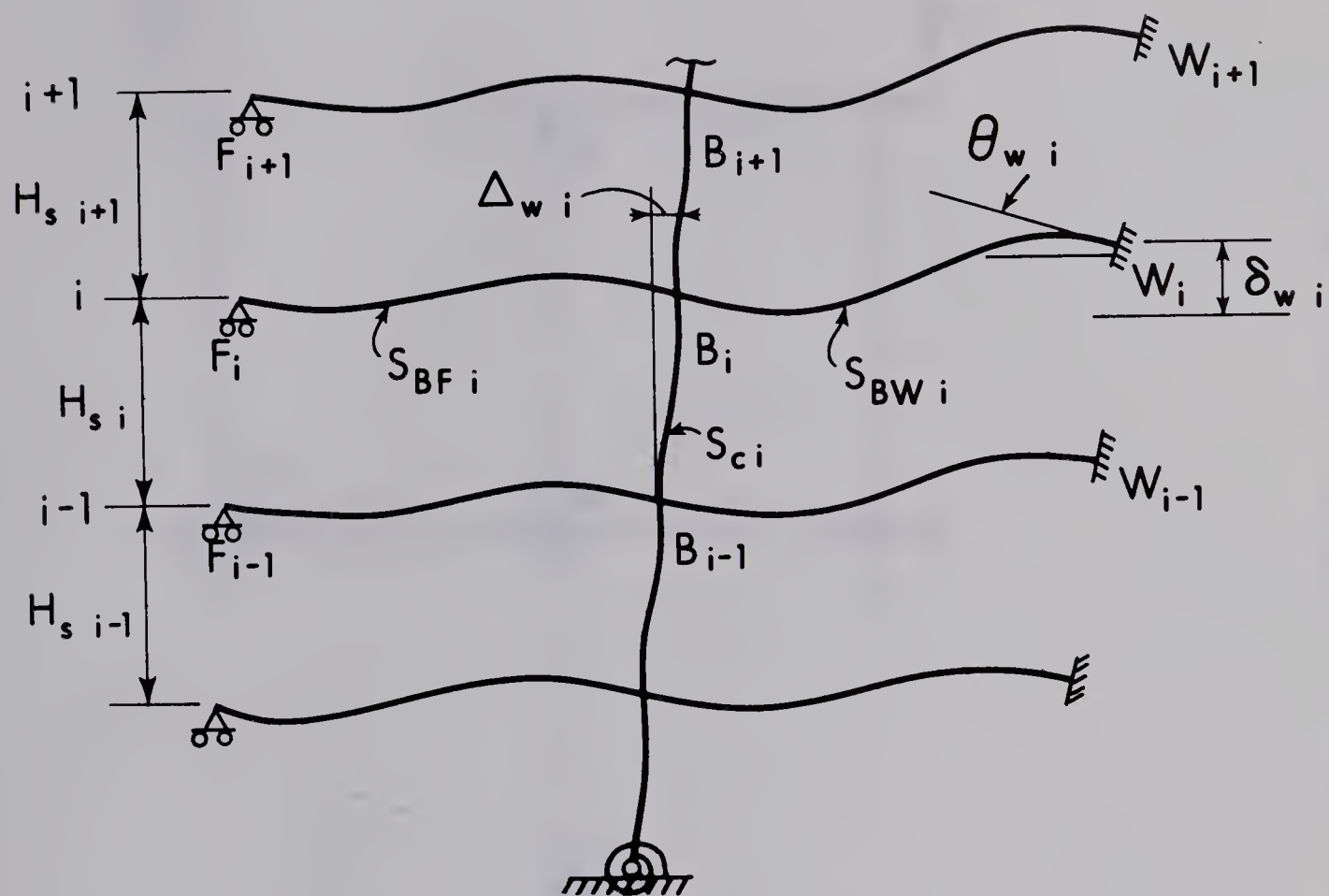


FIGURE 4.5 DEFLECTED FRAME

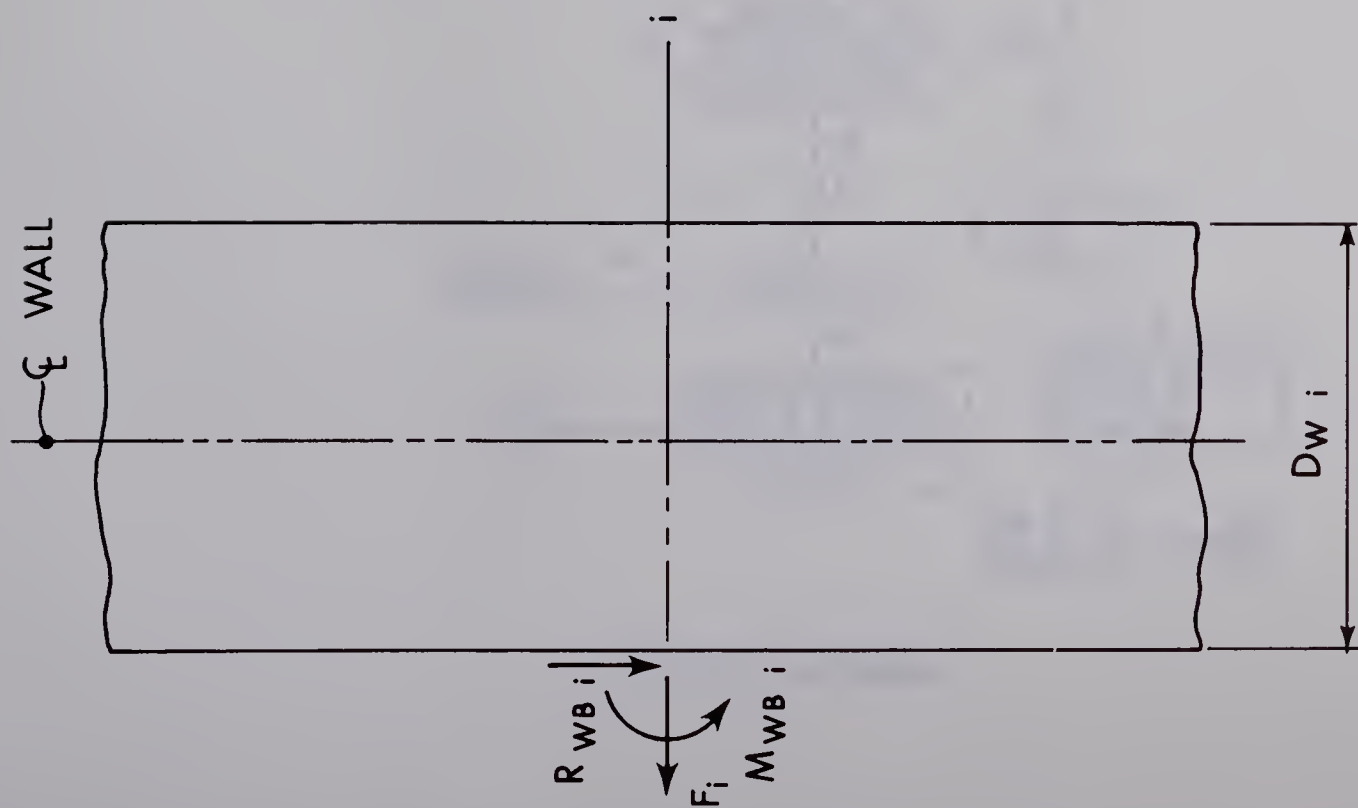


FIGURE 4.6 FRAME FORCES AND MOMENTS ON SHEAR WALL

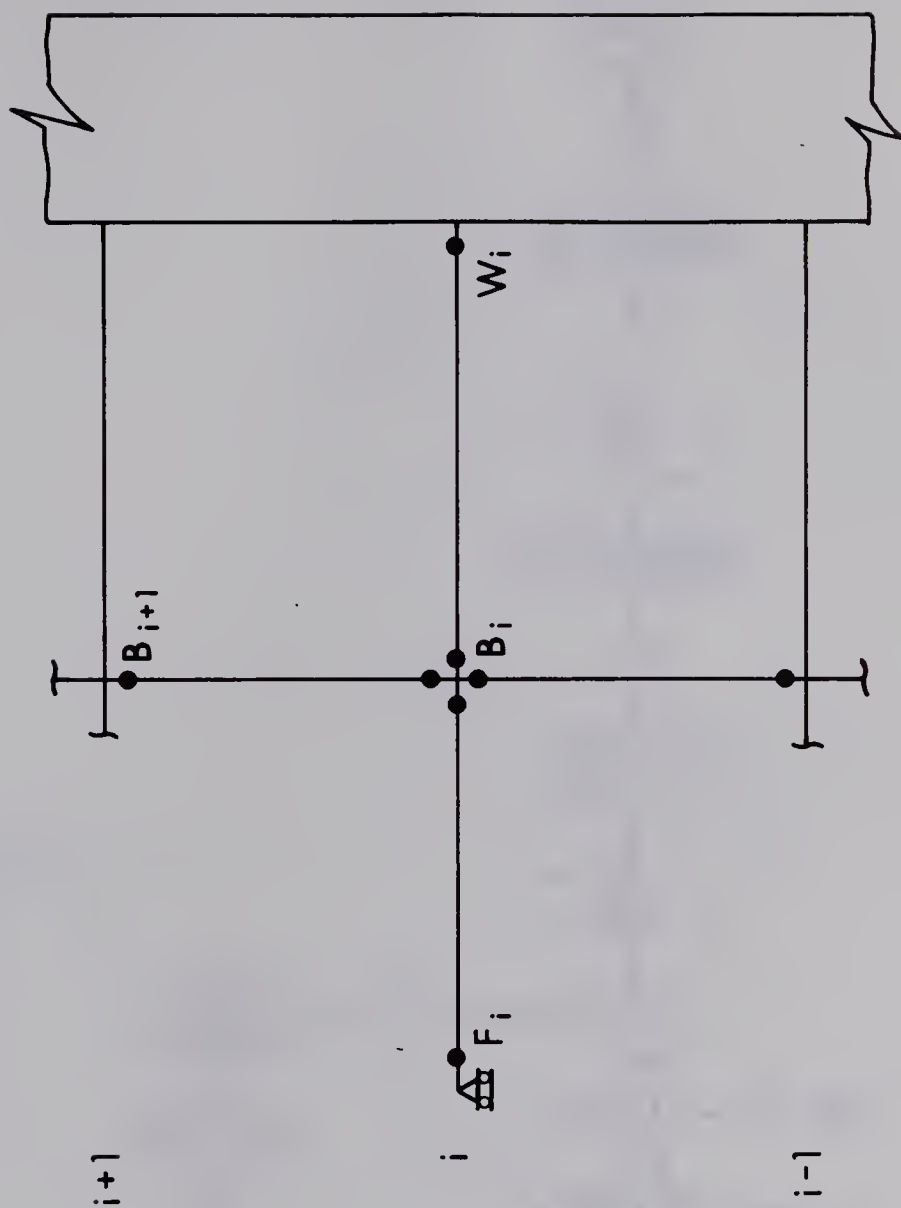


FIGURE 4.7 POTENTIAL PLASTIC HINGE LOCATIONS

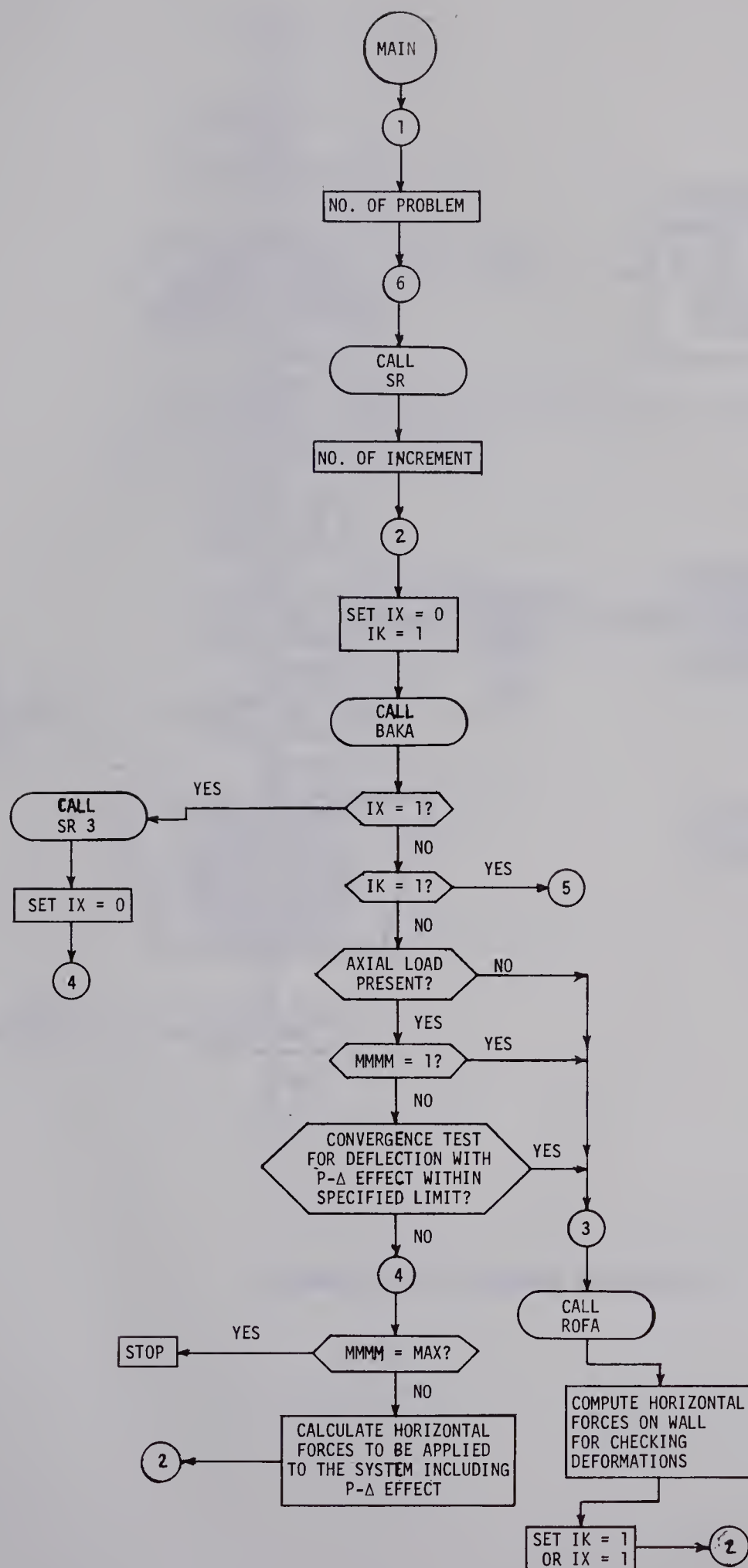


FIGURE 4.8 MAIN PROGRAM

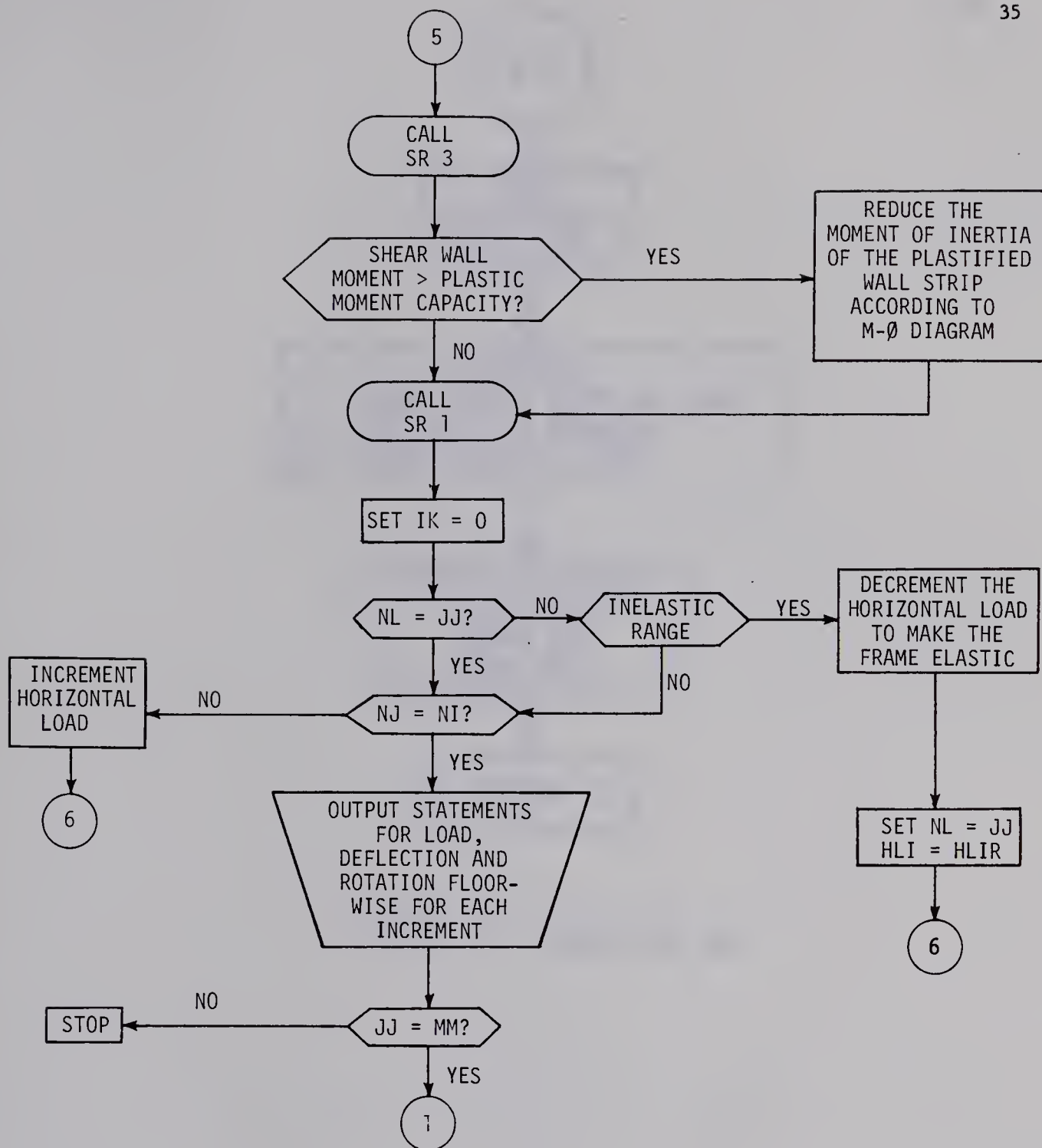


FIGURE 4.8 MAIN PROGRAM (CONTINUED)

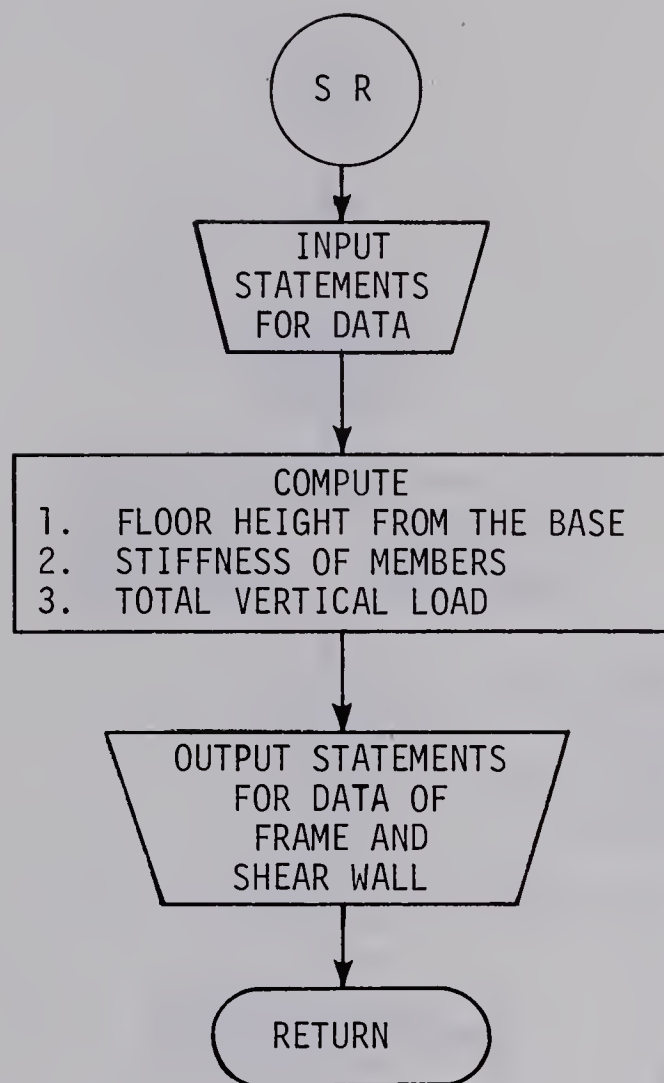


FIGURE 4.9 SUBROUTINE SR

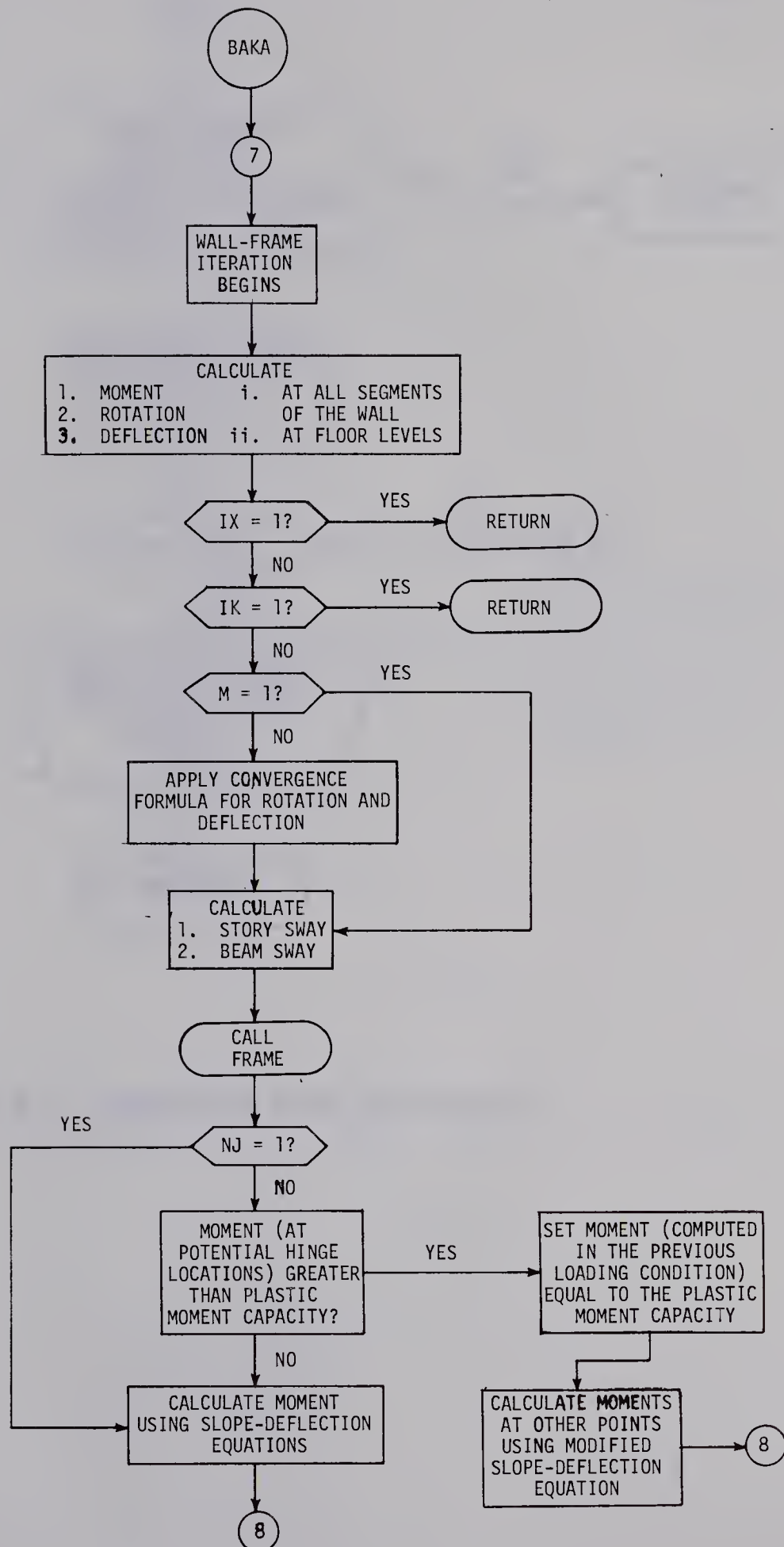


FIGURE 4.10 SUBROUTINE BAKA

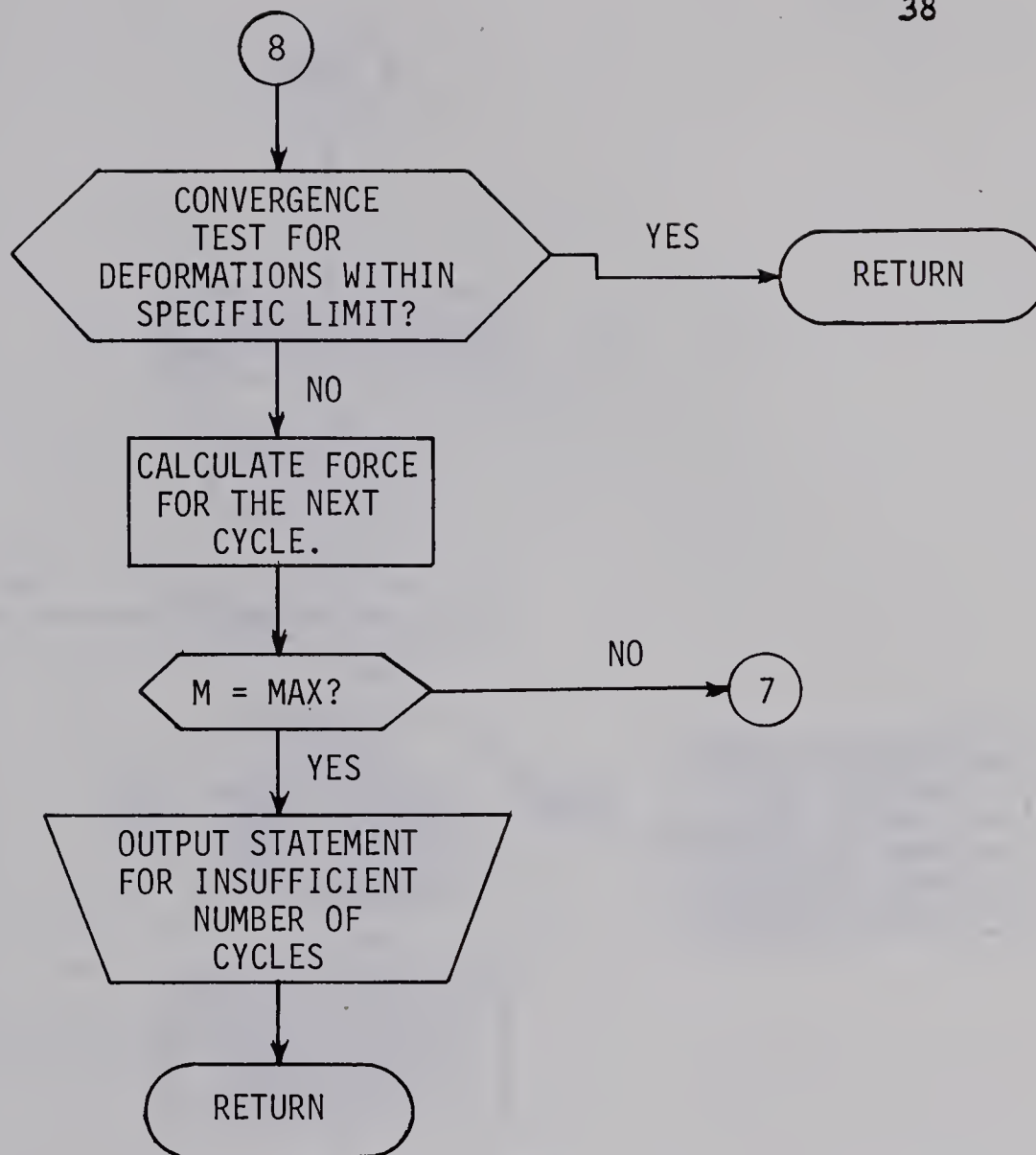


FIGURE 4.10 SUBROUTINE BAKA (CONTINUED)

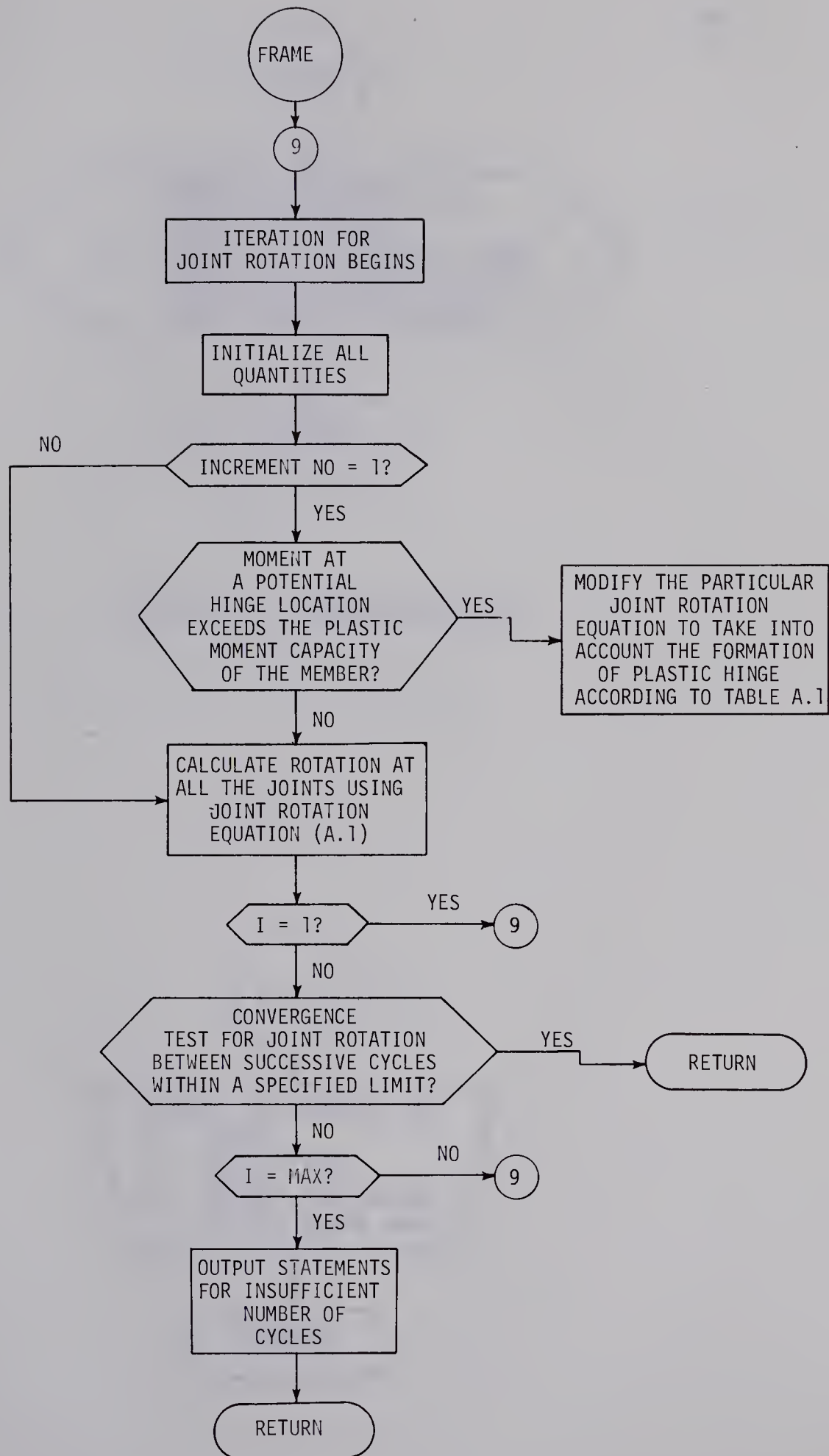


FIGURE 4.11 SUBROUTINE FRAME

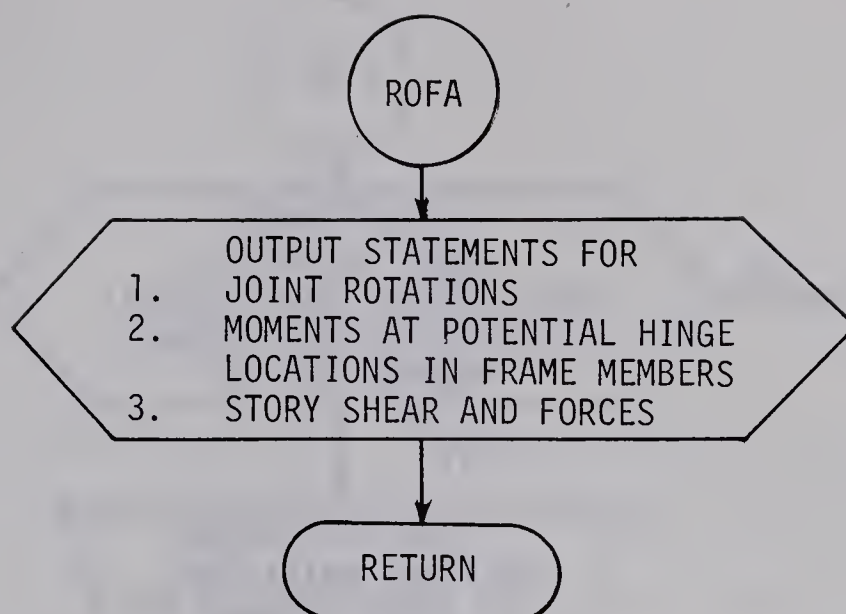


FIGURE 4.12 SUBROUTINE ROFA

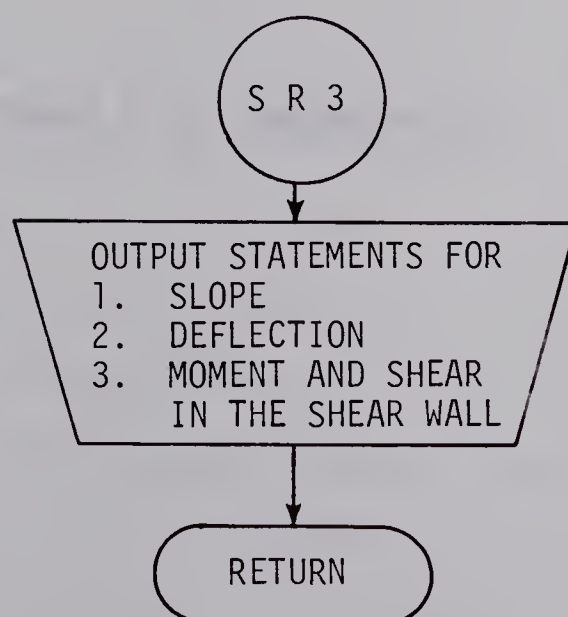


FIGURE 4.13 SUBROUTINE SR 3

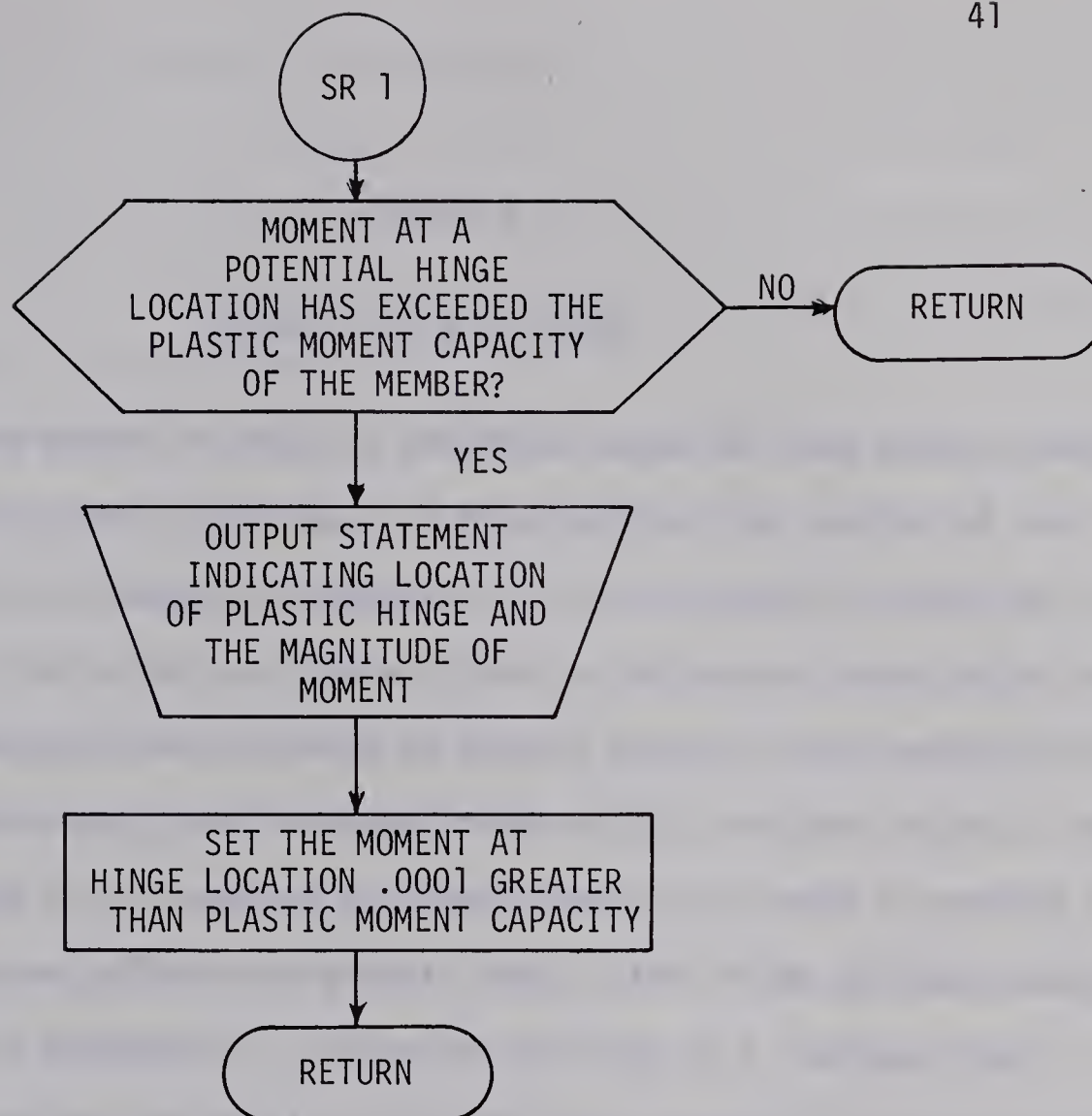


FIGURE 4.14 SUBROUTINE SR 1

CHAPTER V

RESULTS AND DISCUSSION

The method of analysis described above has been used to analyze various multi-story structures. In this section, the results of the analyses of two frames are discussed. The first example is used to demonstrate the validity of the analysis in the elastic range while the second illustrates the influence of several factors on the behavior of a typical multi-story structure both in the elastic and the inelastic range.

The first frame was analyzed primarily in order to provide a check on the analysis in the elastic range. This frame had been analyzed previously in REFERENCE 2. The actual building is a fourteen-story structure, rectangular in plan, with nineteen bays of 11'-6" in the long direction and three bays of 20'-0" in the short direction. The building has been analyzed for a uniformly distributed lateral load applied perpendicular to the long side of the building.

The geometry of the equivalent structure is given in FIGURE 5.1. The equivalent stiffness of the left beam is twice the sum of the stiffnesses of the three beams spanning 20'-0" at a given floor level. The equivalent moments of inertia of the various members (including the shear wall) have been listed in TABLE 5.1. The moments of inertia of the actual members have been given in REFERENCE 2. The structure was analyzed elastically by a different lumping procedure in REFERENCE 2., however, the model used in this report represents a further simplification in that it contains a

single equivalent column. The modulus of elasticity for both the wall and the frame was taken as 3000 k/in^2 . The rotational spring constants at the base of the column and the shear wall were set at relatively large values so that the fixed end condition is approached to represent the original structure.

The distributed lateral load was replaced by single concentrated loads acting at each panel point. In order to obtain values for the vertical loads acting on the structure, reasonable uniformly distributed live and dead loads were assumed. The values of the total vertical load acting at each level of the structure are listed in TABLE 5.1. These values are required for the second-order elastic analysis.

First order and second order (including the $P-\Delta$ effect) elastic analyses were performed on the structure. The results of the first order elastic analysis were compared with the results presented in REFERENCE 2. The agreement was satisfactory. FIGURE 5.2 shows a graph of the story number, N , versus the proportion of the total base shear carried by the frame system, V_f/V_b . The solid lines indicate the results of the first order elastic analysis using the model shown in FIGURE 5.1 and the dashed lines represent the results obtained from REFERENCE 2. The ratio V_f/V_b and thus the actual shear in the story, is relatively constant except near the bottom of the structure. Since the applied shear increases from the top down, the top stories of the frame are forced to carry shears which are much greater than the applied shears. This is due to the pull exerted on the top stories of the frame by the stiff shear wall.

The deflected shape of the fourteen story structure is shown in FIGURE 5.3. This figure plots the story number, N , versus the lateral

deflection, Δ , at the various floor levels. The deflections plotted correspond to a lateral load of 20 psf lumped at each floor level. The solid line represents the deflected shape obtained from a first order elastic analysis. The dotted line shows the deflected shape which results when the P- Δ effect is taken into account. Although the P- Δ effect is not too significant in this example, it becomes important for more flexible structures.

In this example, the shears developed in the top stories were considerably higher than those applied. To study the effect of the wall stiffness on the shear distribution, six additional analyses were performed. Various combinations of shear wall stiffnesses were considered in the top four stories of the structure; the properties of the lower stories remaining constant. In TABLE 5.2, the moments of inertia of the top four stories of the shear wall are listed along with the corresponding shears resisted by the frame system. In this table the frame shears are listed as a percentage of the applied story shears, V_f/V_s . In all cases, the percentage story shear resisted by the top story of the frame was decreased when compared with the results of the original analysis. On the other hand, the percent shear resisted by the lower stories of the frame was increased. For example, considering cases (4) and (5) in TABLE 5.2, there is a decrease in V_f/V_s in the 13th and 14th story and a corresponding increase in the 11th and 12th story.

The second example considered is a twenty-four story, three bay frame. This frame was designed using both the allowable stress and the plastic strength techniques in REFERENCE 20. The general layout of the frame is shown in FIGURE 5.4a. The analytical model used in the present

case is shown in FIGURE 5.4b. The moments of inertia and plastic moment capacities of the beams used in the model structure are listed in TABLE 5.3. The total vertical loads acting on the structure and the properties of the columns are listed in TABLE 5.4. The bracing system shown in FIGURE 5.4a was replaced by a shear wall as shown in FIGURE 5.4b. No attempt was made to obtain a flexural shear wall corresponding to the braced truss; instead several analyses of the structure were performed by varying the flexural stiffness of the shear wall. In all cases, the ratio of the wall stiffness to the column stiffness was constant in each story of the structure.

The plastic moment capacity at the bottom of the wall was selected so to bear a reasonable relationship to the wall stiffness. The plastic moment capacity of the shear wall in the stories above the base was reduced linearly so that the ratio of wall strength to stiffness would be constant for each story. A bilinear moment-curvature relationship was used for the wall with the slope of the strain-hardening branch being one-sixth of the elastic slope. The modulus of elasticity of the frame was $30,000 \text{ kip/in}^2$ and for the shear wall $3,000 \text{ kip/in}^2$. A constant 8'-0" wall width was maintained throughout the height of the structure. High values of the rotational spring constants were used at the base of the column and shear wall in order to approach a fixed end condition. Concentrated lateral loads were assumed to act at each floor level. The lateral load at the top floor was one-half that at the other levels. The vertical load was held constant during the analysis and the lateral loads were increased monotonically.

First and second order (with $P-\Delta$ effect) inelastic analyses were performed for the 24-story structure. FIGURE 5.5 shows graphs of

the lateral force at the top of the frame, H , versus the sway rotation of the top story $(\Delta_{w\ 24} - \Delta_{w\ 23}) / H_{s\ 24}$. The frame has a ratio of wall stiffness to column stiffness of 50. The upper curve has been obtained from a first order inelastic analysis. The first hinge in the frame is detected at point 'a'. The shear wall yields first at the base as shown by point 'b' on the graph. The structure is essentially, a 'weak beam-strong column' type and at point 'c' hinges have formed at the ends of all the beams. The only column hinge detected between points 'b' and 'c' occurs at the top of the column in the 24th story. Since a 'strain-hardening' type of moment-curvature relationship has been assumed for the wall, the wall continues to accept additional load. To demonstrate the $P-\Delta$ effect, an analysis represented by the lower curve in FIGURE 5.5 was performed. In this case, the structure was analyzed with a reduced plastic moment capacity (TABLE 5.4) for the columns. At point 'a' on the lower curve, the first hinge in the structure was detected. Up to point 'b' the rate of decrease of the stiffness of the structure is moderate, however, beyond point 'b' the deflection of the structure increases rapidly. At point 'c' most of the wall is inelastic. For the next increment of lateral load, the deformations become so large that the system does not converge. The load corresponding to 'c' has been taken as the ultimate load carrying capacity of the structure.

In FIGURE 5.5, the effect of the $P-\Delta$ moment on the structure is illustrated. The difference in load carrying capacity predicted by the two analyses is primarily due to the $P-\Delta$ effect, since only one hinge forms in the column between points 'b' and 'c' according to the

first order analysis and hinges did not form in the columns according to the second order analysis.

FIGURE 5.6 consists of plots of the story number, N , versus the lateral deflection, Δ , showing the deflected shape of the structure as the lateral load is incremented. The curves are obtained from the second order analysis in FIGURE 5.5. The values of the applied lateral load at the top of the structure are used to distinguish the curves. The deflected shape for loads of 5.40 kips and 5.60 kips correspond to steps 'b' and 'c' respectively in FIGURE 5.5. These curves emphasize the rapid increase in deflections which occurs as the wall enters the inelastic range.

The shear resisted by the frame in the upper stories is considerably higher than the applied shear. FIGURE 5.7 plots the top level lateral force, H , versus the percentage of the story shear resisted by the frame, V_f/V_s . The curve is obtained from the second order analysis. In FIGURE 5.7, the relationship is almost straight up to point 'a' since few hinges have formed in the frame. From 'a' to 'c' the percentage shear resisted by the frame increases with an increase in the lateral load. Between points 'b' and 'c' the increase is quite rapid. Points 'b' and 'c' in FIGURE 5.7 correspond to the same points, in FIGURE 5.5.

The effect of varying the wall stiffness (keeping the wall width constant) was also studied. FIGURE 5.8 is a plot of top level lateral force, H , versus the top level lateral deflection, Δ . In all cases, the $P-\Delta$ effect (corresponding to the total vertical loads listed in TABLE 5.4) was included and the reduced plastic moment

capacities were used for the columns.

The curves in FIGURE 5.8 represents ratios of the wall to column stiffness ranging from 5 to 50 . This ratio was constant for all stories. The load carrying capacity of the structure decreases as the ratio of the stiffness of the shear wall to the column is decreased.

The structure having $S_w/S_c = 50$ was reanalyzed with zero wall width. The result is shown in FIGURE 5.8 as the dashed curve . The analysis assuming zero wall width neglects the extra restraining moment on the wall. This moment is the result of the shear at the wall end of the right beam acting through half the wall width. For the example considered the analysis predicts a load carrying capacity below that of the actual structure.

It can be observed from FIGURE 5.8 that the difference in behavior due to the variation in wall to column stiffness is considerable. In all cases, the analysis of the shear wall-frame structure did not converge beyond point 'c' . In the analysis, the loads corresponding to points 'c' have been taken as the ultimate load carrying capacities of the structures. Due to the procedure used in the analysis, the unloading branch of the load-deflection curve for the structure can not be obtained.

STORY NO. OR FLOOR NO.	VERTICAL LOADS (kips)	MOMENT OF INERTIA (IN ⁴)		
		BEAM	COLUMN	WALL
1	6.25×10^4	4.66×10^5	9.50×10^5	5.02×10^7
2	5.75×10^4	4.66×10^5	5.96×10^5	5.02×10^7
3	5.25×10^4	4.66×10^5	5.70×10^5	5.02×10^7
4	4.78×10^4	4.58×10^5	5.38×10^5	5.02×10^7
5	4.30×10^4	4.58×10^5	5.07×10^5	5.02×10^7
6	3.83×10^4	4.58×10^5	4.76×10^5	5.02×10^7
7	3.38×10^4	4.50×10^5	4.45×10^5	5.02×10^7
8	2.93×10^4	4.50×10^5	4.16×10^5	5.02×10^7
9	2.48×10^4	4.50×10^5	3.84×10^5	5.02×10^7
10	2.05×10^4	4.42×10^5	3.54×10^5	5.02×10^7
11	1.63×10^4	4.42×10^5	3.24×10^5	5.02×10^7
12	1.20×10^4	4.42×10^5	2.93×10^5	5.02×10^7
13	8.00×10^4	4.34×10^5	2.62×10^5	5.02×10^7
14	4.00×10^4	4.34×10^5	2.32×10^5	5.02×10^7

TABLE 5.1

LOADS AND MEMBER PROPERTIES (FOURTEEN-STORY STRUCTURE)

CASE	MOMENT OF INERTIA OF WALL (STORY) (IN ⁴)					PERCENTAGE STORY SHEAR RESISTED BY FRAME				
	14th	13th	12th	11th	14th	13th	12th	11th	14th	11th
1	5.02×10^7	5.02×10^7	5.02×10^7	5.02×10^7	692	182	123	91		
2	5.02×10^6	5.02×10^7	5.02×10^7	5.02×10^7	650	190	110	114		
3	5.02×10^5	5.02×10^7	5.02×10^7	5.02×10^7	412	212	109	116		
4	2.32×10^5	5.02×10^7	5.02×10^7	5.02×10^7	304	240	110	118		
5	5.02×10^5	5.02×10^5	5.02×10^7	5.02×10^7	252	153	139	120		
6	5.02×10^6	5.02×10^6	5.02×10^6	5.02×10^7	495	152	108	122		
7	5.02×10^5	5.02×10^5	5.02×10^6	5.02×10^6	238	140	120	120		

TABLE 5.2 EFFECT OF WALL STIFFNESS REDUCTION

(FOURTEEN-STORY STRUCTURE)

LEVEL	MOMENT OF INERTIA (IN ⁴)		PLASTIC MOMENT CAPACITY (KIP-IN)	
	BEAM AB	BEAM BC	BEAM AB	BEAM BC
1	888	1170	5404	4920
2	888	1170	5404	4920
3	888	1170	5404	4920
4	888	1170	5404	4920
5	888	984	5404	4560
6	888	984	5404	4560
7	888	984	5404	4560
8	888	984	5404	4560
9	888	890	5404	4020
10	888	890	5404	4020
11	888	890	5404	4020
12	888	890	5404	4020
13	888	890	5404	4020
14	888	890	5404	4020
15	888	890	5404	4020
16	888	890	5404	4020
17	888	890	5404	4020
18	888	890	5404	4020
19	888	890	5404	4020
20	888	890	5404	4020
21	888	890	5404	4020
22	888	890	5404	4020
23	888	890	5404	4020
24	630	584	3900	2960

TABLE 5.3 BEAM PROPERTIES (TWENTY FOUR-STORY STRUCTURE)

LEVEL OR STORY	COLUMN PROPERTIES		VERTICAL LOAD (KIPS)	REDUCED PLASTIC MOMENT-CAPACITY OF COLUMN (KIP-IN)
	MOMENT OF INERTIA (IN ⁴)	PLASTIC MOMENT CAPACITY (K-IN)		
1	14814	73000	12680	27300
2	14814	73000	12330	27300
3	13278	66100	11600	25400
4	13278	66100	11000	25400
5	11452	58000	10540	21900
6	11452	58000	10000	21900
7	9716	50600	9440	18400
8	9716	50600	8930	18400
9	8306	43600	8400	15750
10	8306	43600	7860	15750
11	7098	37800	7320	13500
12	7098	37800	6790	13500
13	5850	31600	6260	11500
14	5850	31600	5720	11500
15	4850	26600	5180	10800
16	4850	26600	4650	10800
17	4130	22900	4120	10300
18	4130	22900	3590	10300
19	2970	16700	3050	6960
20	2970	16700	2520	6960
21	1620	10500	1980	5580
22	1620	10500	1450	5580
23	1100	6060	1050	5260
24	1100	6060	370	5260

TABLE 5.4 VERTICAL LOADS AND COLUMN PROPERTIES
(TWENTY FOUR-STORY STURCTURE)

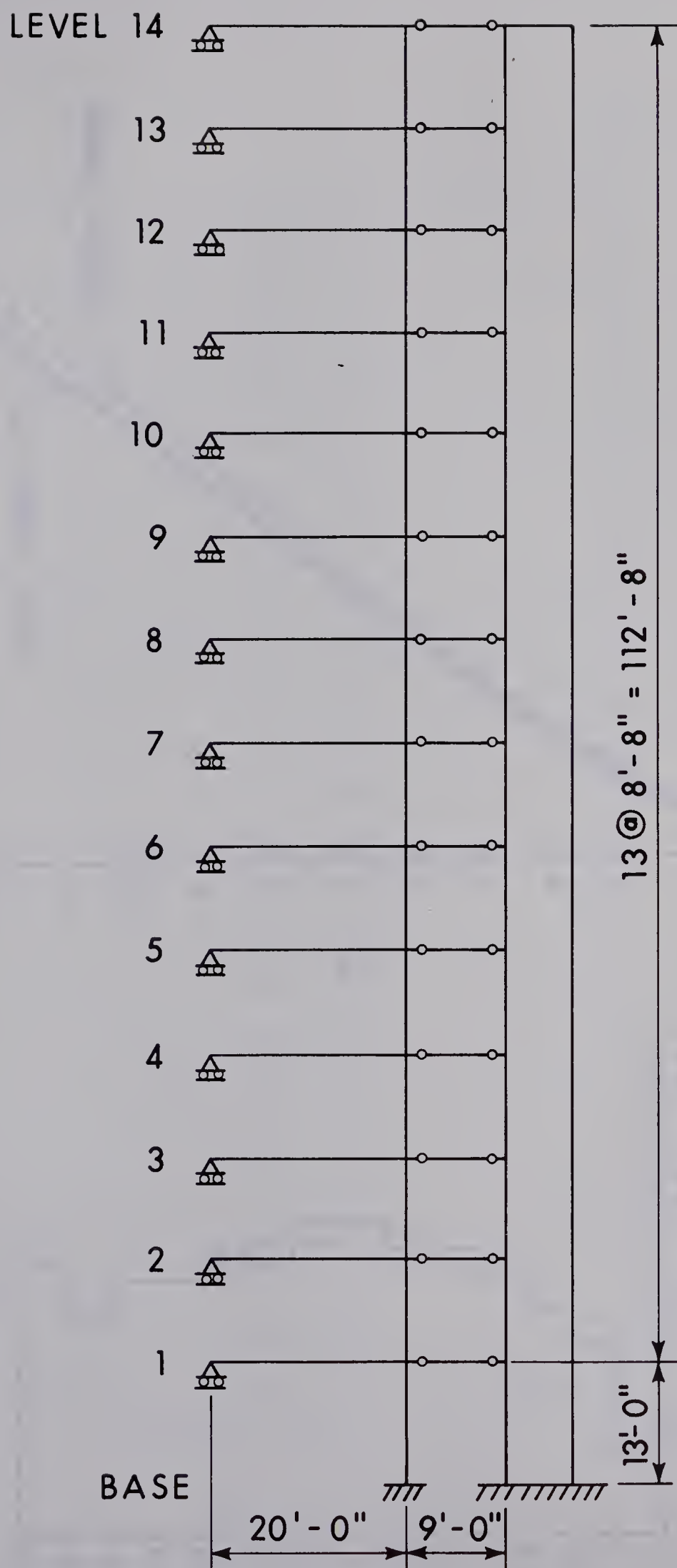


FIGURE 5.1 FOURTEEN-STORY STRUCTURE

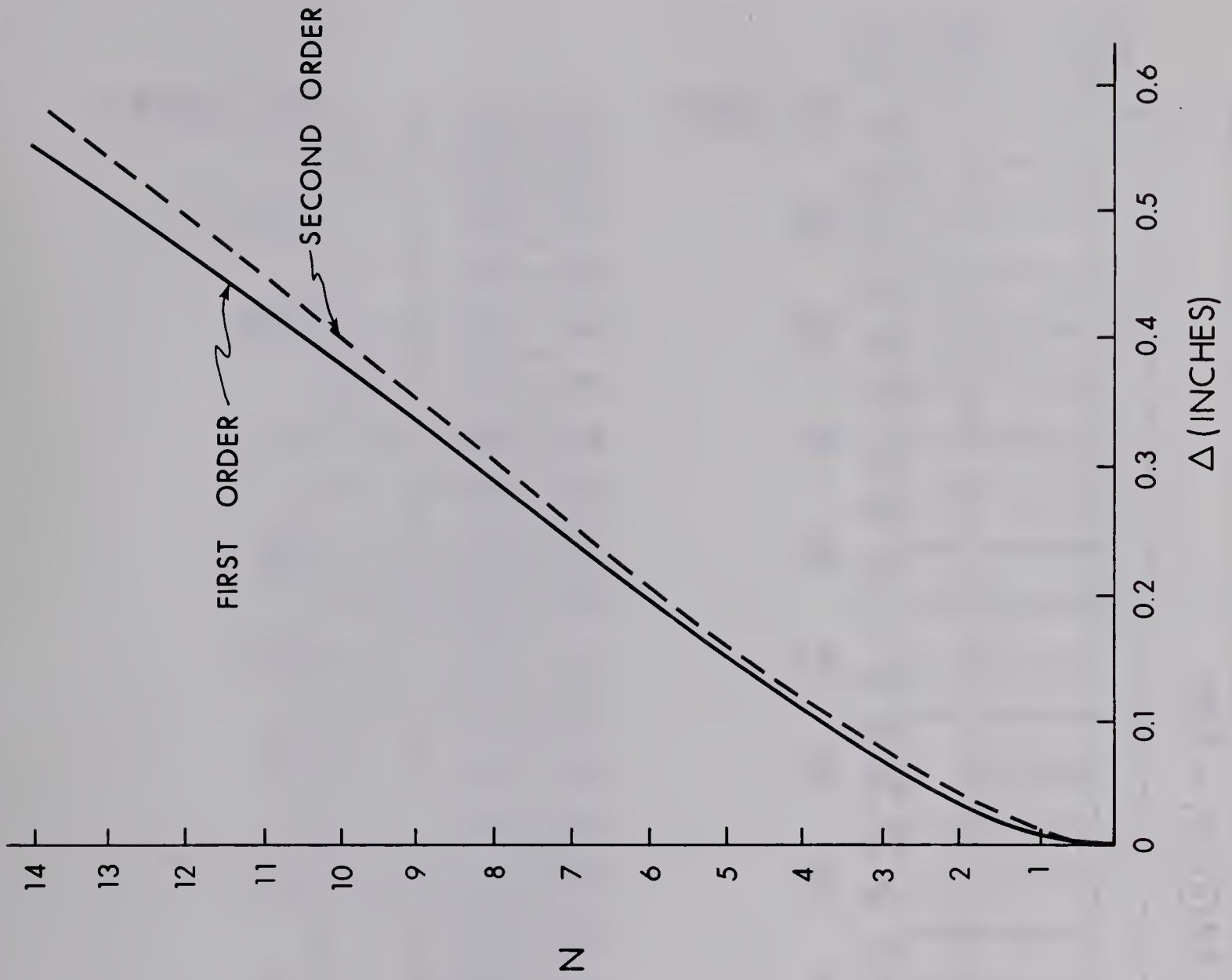


FIGURE 5.3 DEFLECTED SHAPE OF FOURTEEN-STORY STRUCTURE

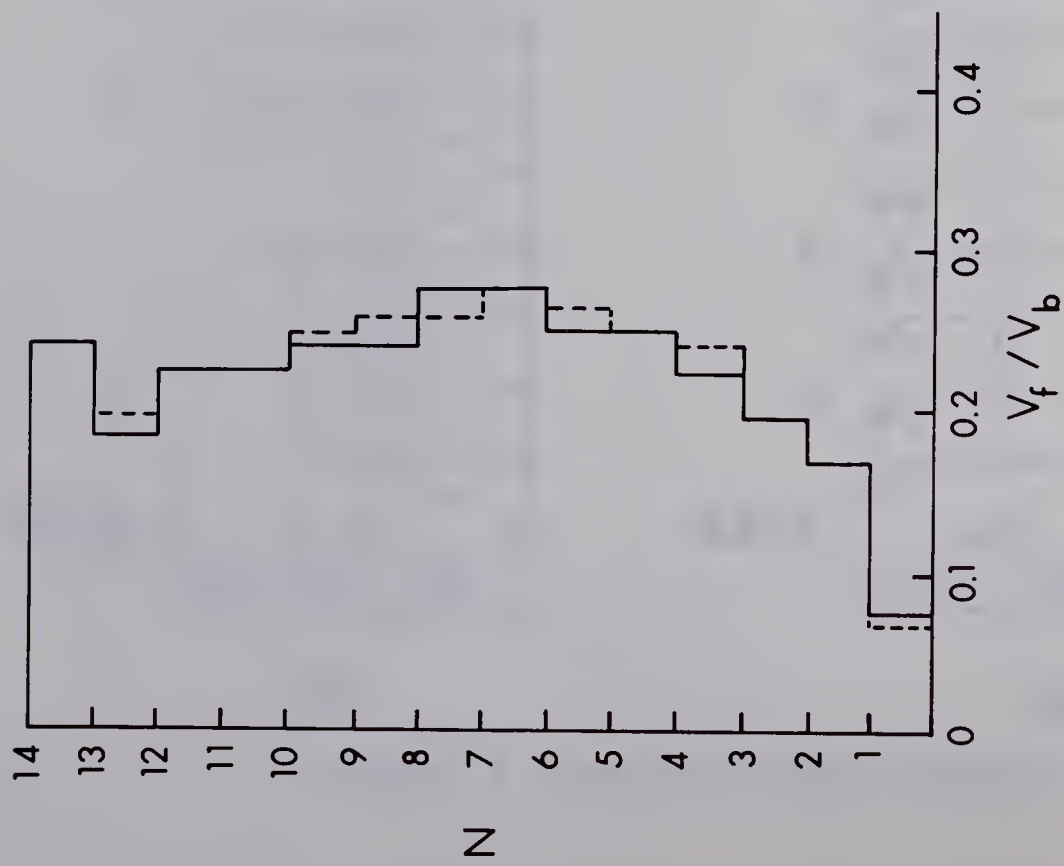
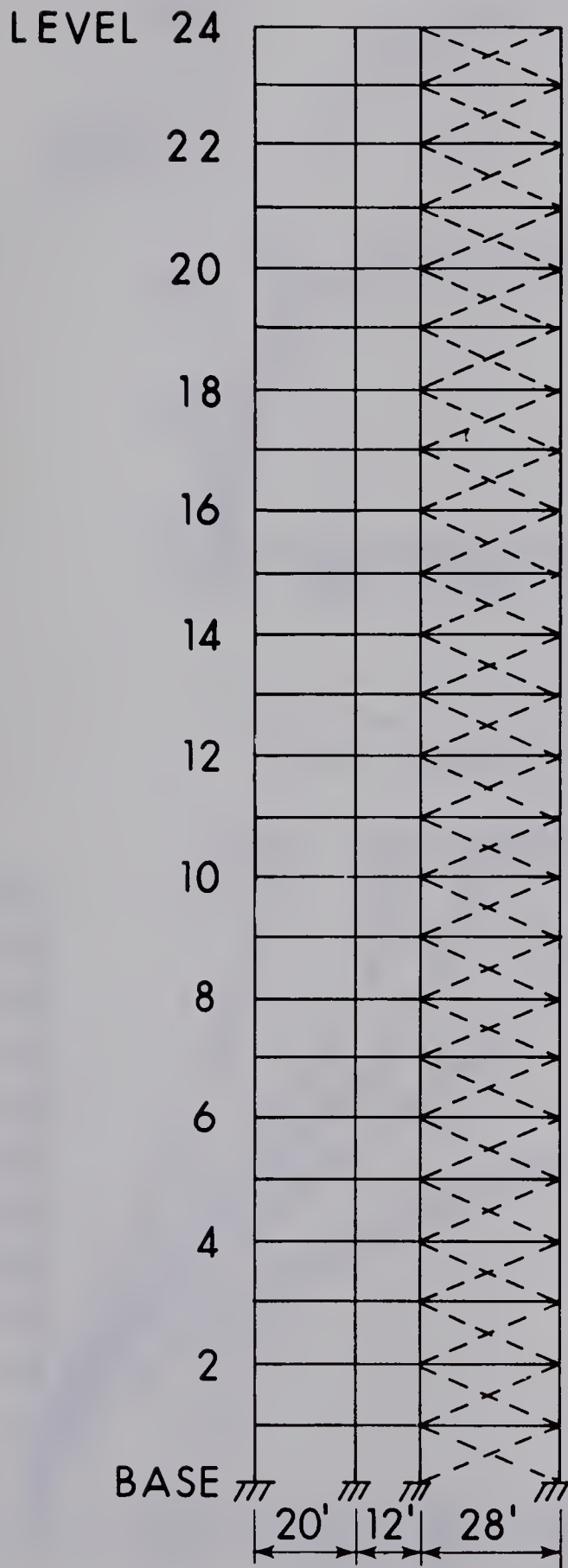
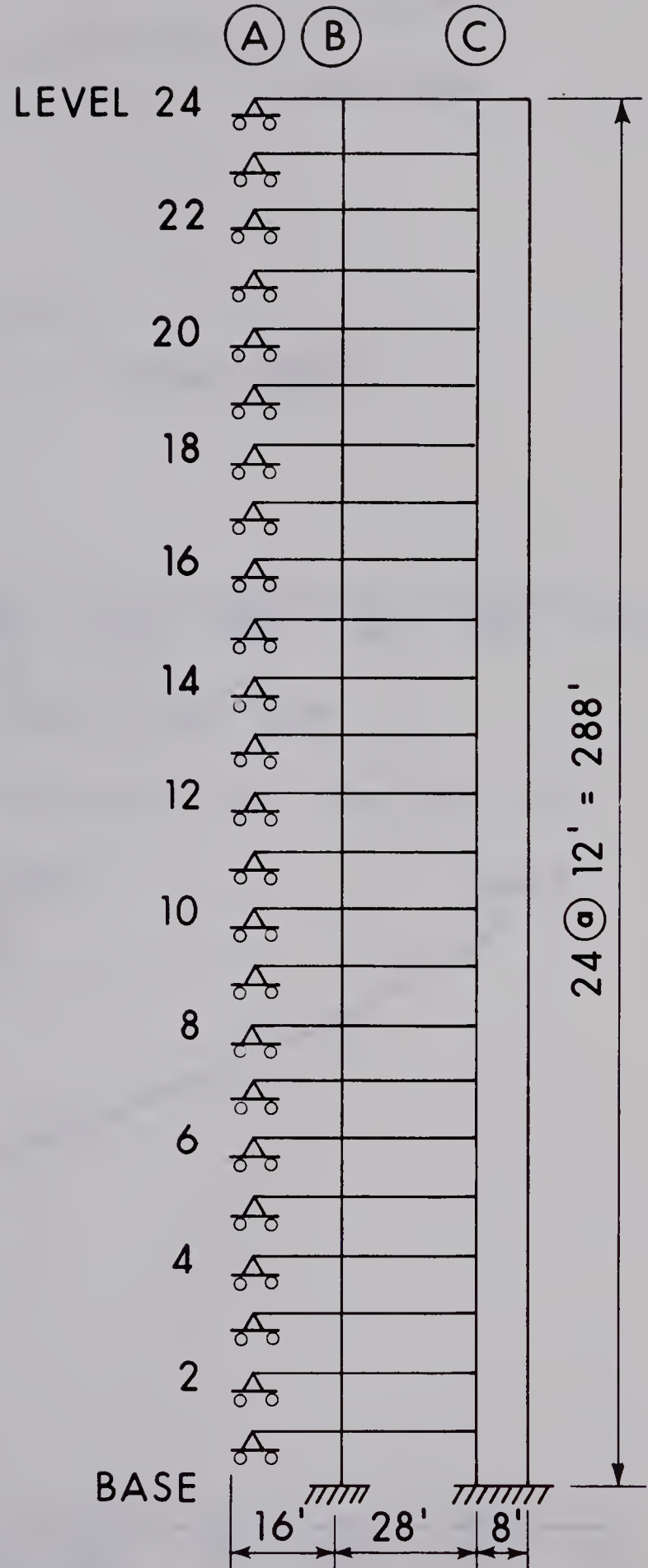


FIGURE 5.2 STORY VS. RATIO OF STORY SHEAR TO BASE SHEAR



(a)



(b)

FIGURE 5.4 TWENTY-FOUR-STORY STRUCTURE

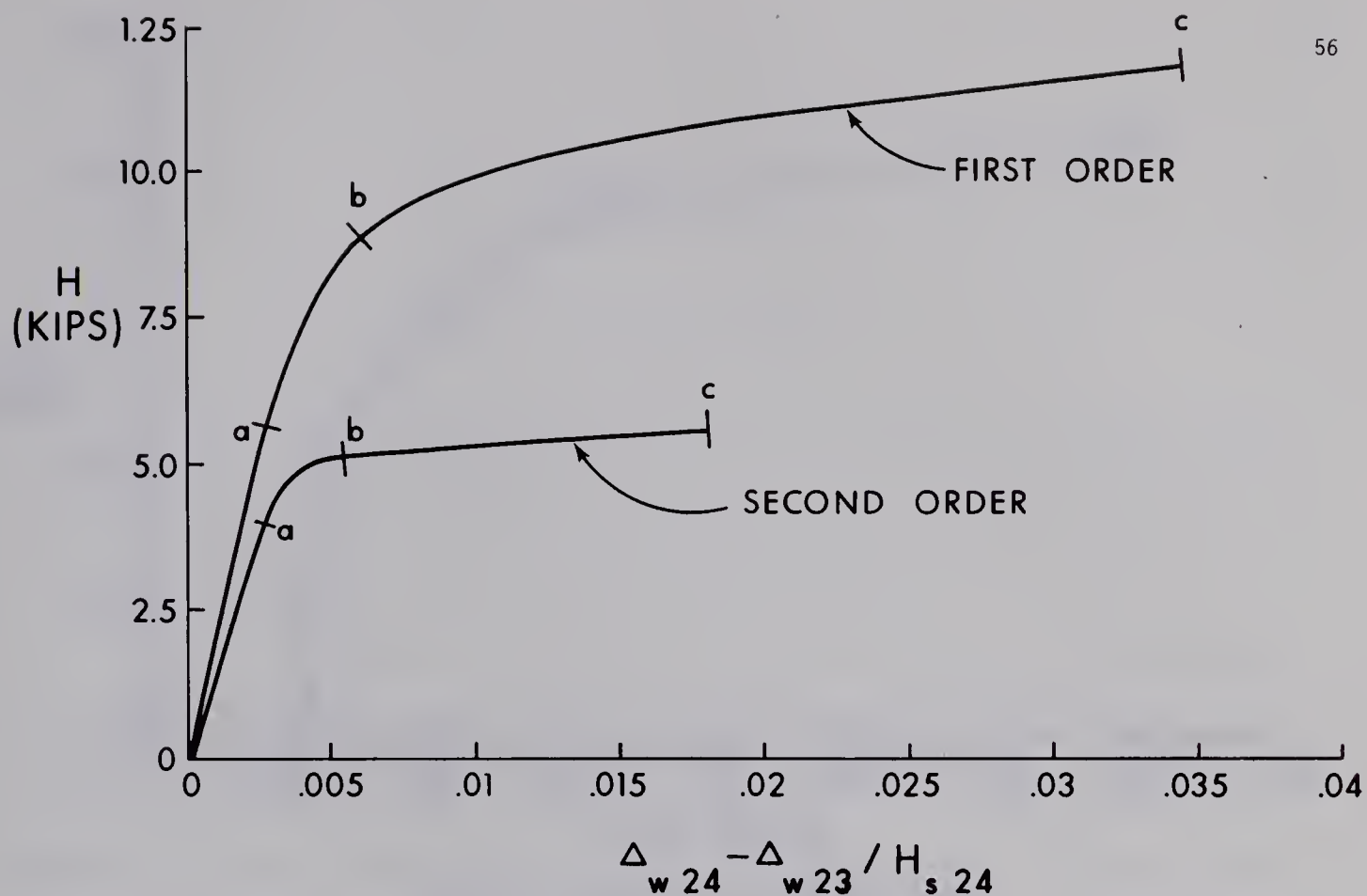


FIGURE 5.5 TOP LEVEL LOAD - TOP STORY SWAY CURVE (WALL TO COLUMN STIFFNESS - 50)

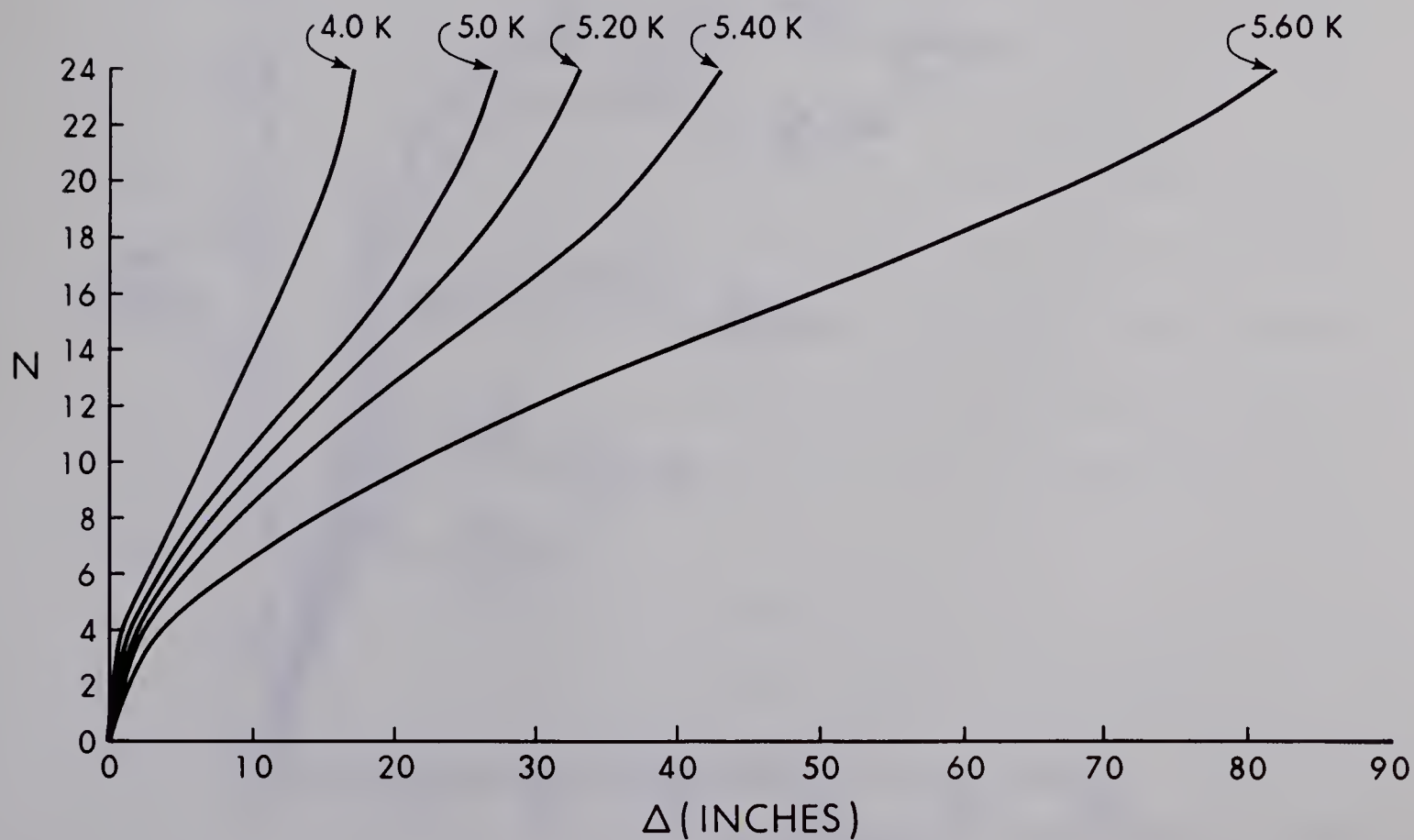


FIGURE 5.6 DEFLECTED SHAPE OF TWENTY-FOUR-STORY STRUCTURE (WALL TO COLUMN STIFFNESS - 50)

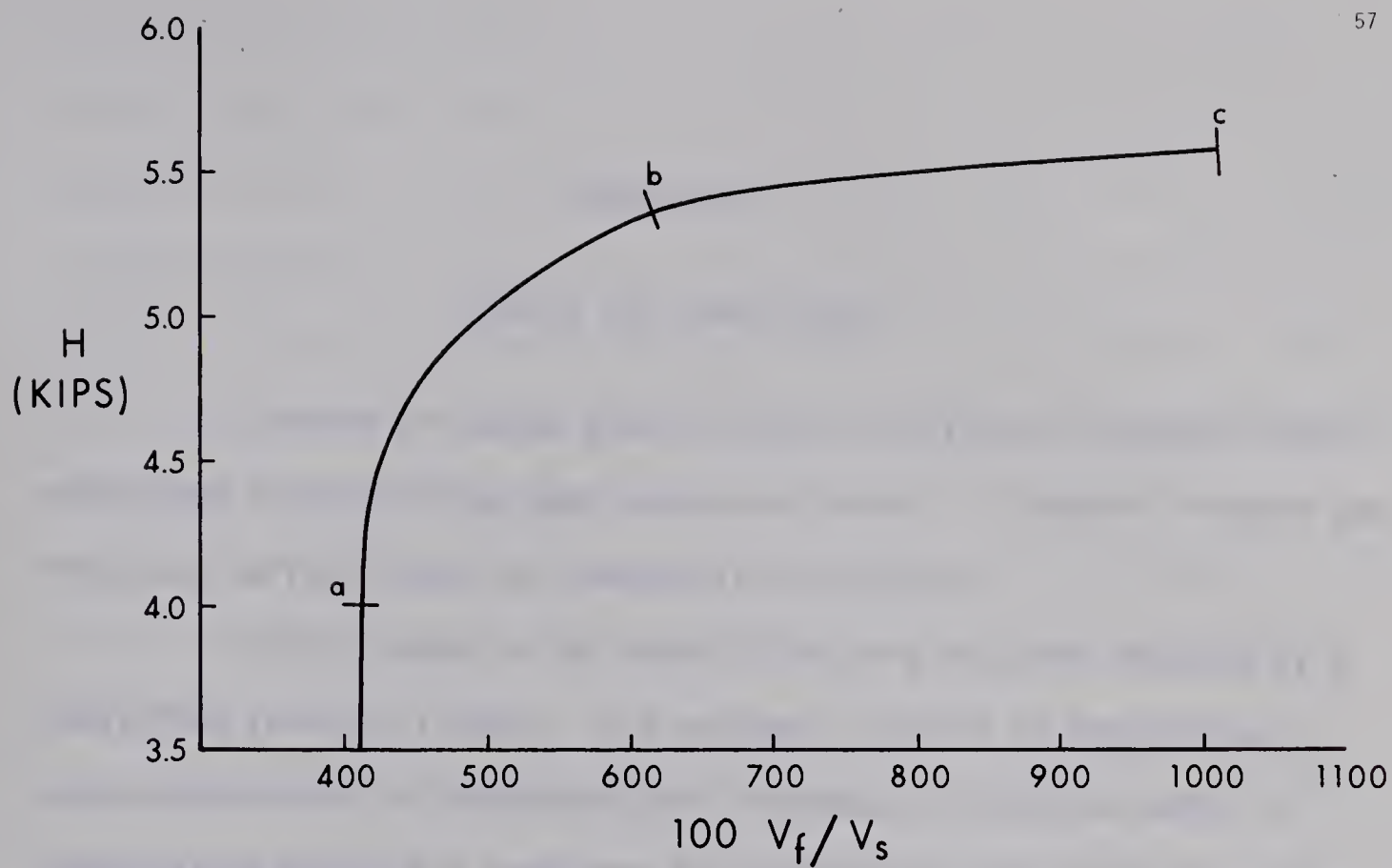


FIGURE 5.7 VARIATION OF TOP STORY FRAME SHEAR

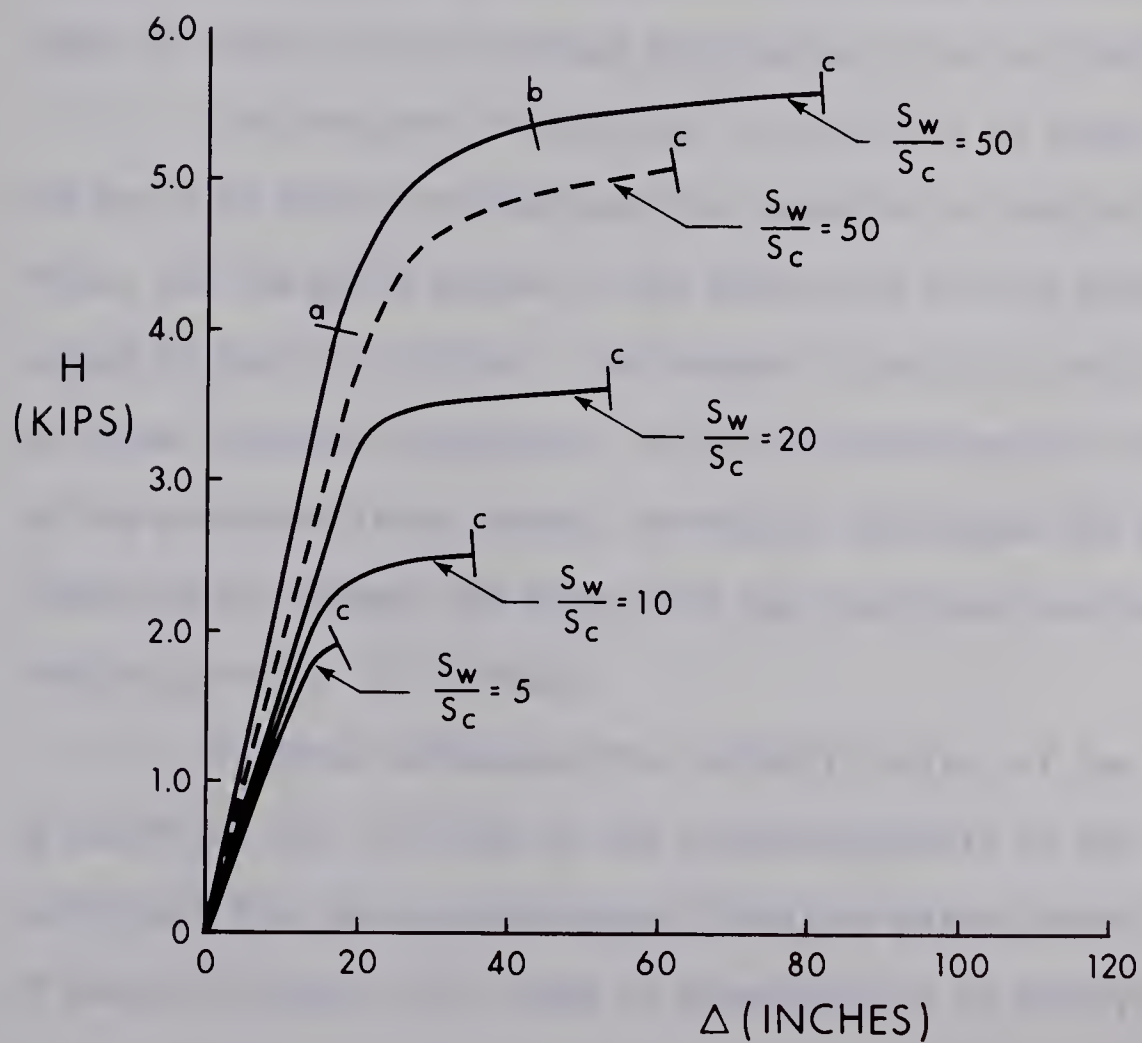


FIGURE 5.8 TOP LEVEL LOAD-DEFLECTION CURVE (TWENTY-FOUR-STORY STRUCTURE)

CHAPTER VI

SUMMARY AND CONCLUSIONS

A method of second order inelastic analysis of combined shear wall-frame structures has been presented herein. A computer program was developed to facilitate the computations involved.

In this analysis the actual structure has been replaced by a simplified structural model. The problems involved in determining a practicable model are discussed with reference to previous work. A conservative method has been used for lumping the actual structure into the model. However, the justification for using this method for various types of shear wall-rigid frame structures is not yet fully developed.

The analysis of the model is discussed in detail. The model was analyzed taking into account the formation of plastic hinges in the frame, the inelastic action of the shear wall and the secondary moments caused by the $P-\Delta$ effect. The method of analysis consists basically of three iterative processes: the first determines the joint rotations of the displaced frame system; the second determines the distribution of lateral shear between the shear wall and the frame; and the third takes into account the $P-\Delta$ effect.

In these processes, the inelastic action of the wall is included by modifying the stiffness of the yielded segments of the wall so as to be consistent with the assumed moment-curvature relationship. The formation of plastic hinges in the frame is accounted for by modifying the joint

rotation equations at the pertinent locations. After the formation of a plastic hinge in the frame, the analysis is continued by assuming that an imaginary hinge is inserted at the cross-section; this hinge is subjected to opposing moments equal to the plastic moment capacity of the section. The $P-\Delta$ effect has been simulated by subjecting the analytical model to an equivalent lateral force system, which is increased to account for the presence of the secondary moments.

A printout of the computer program is given in APPENDIX C. The results of the analyses of two example frames have been presented and discussed.

It is concluded that the analytical model yields comparable results (in the elastic range) to those obtained by using the method developed previously ⁽²⁾. The model is relatively simple and gives a good indication of the behavior of the structure. The rationality of the lumping procedure used and the behavior of the structure in the inelastic range are still to be checked by more rigorous analysis and tests.

The analyses performed have clearly shown the necessity of considering the interaction between the frame and the shear wall. This interaction must be evaluated if the actual force distribution in the structure is to be estimated. Generally in tall structures, the shears developed in the top stories are much higher than the applied story shears. Neglect of this factor could lead to excessive deflections and possible failure of the affected frame members.

The $P-\Delta$ effect caused a considerable reduction in load carrying capacity for the more flexible structure analyzed. The secondary moments produced by the $P-\Delta$ effect, combined with the inelastic action

of the wall and frame resulted in reduced load-carrying capacities. The frames analyzed did not form a mechanism at the ultimate load but instead failure was due to overall instability of the structure.

The vertical deflections of the beam-to-column joints due to axial column shortening have not been included in this analysis. This effect may influence the deformations of the structure resulting in an additional reduction in the ultimate load carrying capacity.

Although the analysis has been checked by manual computations on simple structures, it is recommended that the rationality of the assumptions made be verified by the following three steps: first, the subassemblage analysis should be checked against a rigorous analysis performed by using an independent program, secondly, the actual structure should be analyzed rigorously and the results compared with those of a subassemblage analysis, thirdly, the predicted behavior should be checked by performing full scale tests.

Due to the method of analysis used, it is not possible to obtain the unloading branch of the load-deflection curve. In order to understand thoroughly the behavior of the structure, the unloading branch must be determined.

NOMENCLATURE

$D_{W i}$	Width of shear wall system at the i th floor
E	Youngs modulus of elasticity
F_i	Applied force on the wall at the i th floor from the frame analysis
$F_{W i}$	Portion of the applied lateral load resisted by the wall at the i th floor
H_i	Applied lateral load on the structure at the i th floor
$H'_1 i$	Additional lateral load to be applied at the i th floor in the first cycle to simulate the $P-\Delta$ effect on the structure
$H_{s i}$	Story height in the i th story
K_{12}	End moment of the beam (12) at the end (1) for a unit rotation at the end (2)
K_B	Stiffness of one beam
K_{BE}	Equivalent beam restraint
K_C	Stiffness of one column
K_{CE}	Equivalent column stiffness
L_{12}	Length of beam (12)
M_p	Plastic moment capacity of shear wall
M_{pc}	Plastic moment capacity of column reduced for axial load
$M_{BF i}$	Moment at the column end of the left hand beam at the i th floor

$M_{BW\ i}$	Moment at the column end of the right hand beam at the i th floor
$M_{c\ i}$	Moment at the top of the column of the i th story
$M_{c\ i+1}$	Moment at the bottom of the column of the $i+1$ th story
$M_{W\ i}$	Total moment on the wall at the i th floor from the frame analysis
$M_{WB\ i}$	Moment at the wall end of the right hand beam at the i th floor
$MP_{BF\ i}$	Plastic moment capacity of the left hand beam at the i th floor
$MP_{BW\ i}$	Plastic moment capacity of the right hand beam at the i th floor
$MP_{c\ i}$	Plastic moment capacity (reduced for axial load) of the column of the i th story
P_i	Total vertical load on the structure above the i th floor
R_i	Chord rotation of the column of the i th story
R'_i	Vertical chord rotation of the right hand beam connecting the column to the wall at the i th floor
$S_{BF\ i}$	Stiffness (EI/L) of the left hand beam at the i th floor
$S_{c\ i}$	Stiffness (EI/L) of the column of the i th story
V'_i	Extra shear in the structure at the i th story due to $P-\Delta$ effect
$\Delta_l\ i$	Final deflection of the structure under the lateral load system H_i at the i th floor. The subscript l refers to the first cycle in the $P-\Delta$ iteration process

Δ_{2i}	Final deflection of the structure under the lateral load system $H_i + H_1' i$ at the i th floor
$\Delta_{WI}^{(n)}$	Lateral displacement of the wall at the i th floor in the n th cycle of the wall-frame iteration process
$\Delta_{WI}'^{(n)}$	Lateral displacement of the wall at the i th floor after applying forced convergence formula in the n th cycle of the wall-frame iteration process
θ_i	Rotation of the frame joint at the i th floor
θ_{Fi}	Rotation at the left end of the left hand beam at the i th floor
$\theta_{Wi}^{(n)}$	Slope of the wall at the i th floor in the n th cycle of the wall-frame iteration process
$\theta_{Wi}'^{(n)}$	Slope of the wall at the i th floor after applying forced convergence formula in the n th cycle of the wall-frame iteration process
$\theta_{WB i}$	Rotation at the wall end of the right hand beam at the i th floor
ϕ_{pc}	Curvature corresponding to M_{pc} assuming ideally elastic behavior ($\phi_p = M_p/EI$)
$\delta_{WI}^{(n)}$	Vertical deflection of the beam-to-column connection at the i th floor, in the n th cycle of the wall-frame iteration process
$\delta_{WI}'^{(n)}$	Vertical deflection of the beam-to-wall connection at the i th floor in the n th cycle of the wall-frame iteration process

NOMENCLATURE FOR FORTRAN IV PROGRAM

BAKA	A SUBROUTINE SUBPROGRAM
BMOM	BOTTOM MOMENT IN A SEGMENT OF THE SHEAR WALL (KIP-IN)
BMOMF	BOTTOM MOMENT IN THE BOTTOM OF THE LOWEST SEGMENT IN A STORY (KIP-IN)
CON	CONVERGENCE LIMIT FOR THE DEFLECTION AND ROTATION
CDEFF	FLOOR LEVEL DEFLECTION OF SHEAR WALL AFTER APPLYING CONVERGENCE FORMULA
CROTF	FLOOR LEVEL ROTATION OF SHEAR WALL AFTER APPLYING CONVERGENCE FORMULA
DEF	DEFLECTION OF WALL AT A SECTION (IN)
DEFF	DEFLECTION OF THE WALL AT THE FLOOR LEVEL (IN)
DMBF	MOMENT AT THE COLUMN END OF THE LEFT HAND BEAM COMPUTED IN THE PREVIOUS LOADING CONDITION (KIP-IN)
DMBW	MOMENT AT THE COLUMN END OF THE RIGHT HAND BEAM COMPUTED IN THE PREVIOUS LOADING CONDITION (KIP-IN)
DMCB	MOMENT AT THE BOTTOM END OF COLUMN IN A PARTICULAR STORY COMPUTED IN THE PREVIOUS LOADING CONDITION (KIP-IN)
DMCT	MOMENT AT THE TOP END OF A COLUMN IN A PARTICULAR STORY COMPUTED IN THE PREVIOUS LOADING CONDITION (KIP-IN)
DMFB	MOMENT AT THE LEFT END OF THE LEFT HAND BEAM COMPUTED IN THE PREVIOUS LOADING CONDITION (KIP-IN)
DMWB	MOMENT AT THE WALL END OF THE RIGHT HAND BEAM COMPUTED IN THE PREVIOUS LOADING CONDITION (KIP-IN)

DW	WIDTH OF SHEAR WALL (IN)
EF	YOUNGS MODULUS OF ELASTICITY OF THE FRAME (KIP/IN ²)
EW	YOUNGS MODULUS OF ELASTICITY OF THE WALL (KIP/IN ²)
F	APPLIED FORCE ON THE WALL FROM FRAME ANALYSIS (KIP)
FD	HORIZONTAL DESIGN LOAD ACTING AT FLOOR LEVEL (KIP)
FRAME	A SUBROUTINE SUBPROGRAM
FOC	FRAME FORCE AT THE FLOOR LEVEL (KIP)
FW	FINAL FORCE ON WALL AT EACH FLOOR LEVEL (KIP)
HBF	LENGTH OF LEFT HAND BEAM (IN)
HBW	LENGTH OF RIGHT HAND BEAM (IN)
HLI	PERCENTAGE OF ORIGINAL HORIZONTAL LOAD TO BE INCREASED IN THE ELASTIC RANGE
HLIR	PERCENTAGE OF ORIGINAL HORIZONTAL LOAD TO BE INCREASED IN THE INELASTIC RANGE
HS	STORY HEIGHT (IN)
HSF	HEIGHT OF FLOOR LEVEL FROM BASE (IN)
IK , IX	DUMMY CONSTANTS
IVL	DUMMY CONSTANTS FOR STOPPING THE PROGRAM IF THE DEFORMATION EXCEEDS CERTAIN SPECIFIED LIMIT
KB	SPRING CONSTANT AT THE BASE OF THE SHEAR WALL (KIP-IN/RAD)
KC	SPRING CONSTANT AT THE BASE OF THE SHEAR WALL (KIP-IN/RAD)
L	PRODUCT OF NUMBER OF STORY AND NUMBER OF SEGMENTS IN A STORY
MAX	MAXIMUM NO. OF CYCLE TO BE PERFORMED FOR ANY ITERATION PROCESS
MI	MOMENT OF INERTIA OF SHEAR WALL (IN ⁴)
MIBF	MOMENT OF INERTIA OF LEFT HAND BEAM (IN ⁴)

MIBW	MOMENT OF INERTIA OF RIGHT HAND BEAM (IN^4)
MIC	MOMENT OF INERTIA OF COLUMN (IN^4)
MII	MOMENT OF THE SEGMENT OF THE SHEAR WALL (IN^4)
MM	NO. OF PROBLEMS TO BE SOLVED
MMCB	MOMENT AT THE BOTTOM OF A COLUMN IN A STORY (KIP-IN)
MMCT	MOMENT AT THE TOP OF A COLUMN IN A STORY (KIP-IN)
MMMM	COUNTER FOR ITERATION WITH THE EFFECT OF AXIAL LOAD
MOMB	MOMENT AT THE BASE OF THE SHEAR WALL (KIP-IN)
MOMBF	MOMENT AT THE COLUMN END OF THE LEFT HAND BEAM IN THE CYCLE UNDER CONSIDERATION (KIP-IN)
MOMBW	MOMENT AT THE COLUMN END OF THE RIGHT HAND BEAM IN THE CYCLE UNDER CONSIDERATION (KIP-IN)
MOMFB	MOMENT AT THE LEFT END OF THE LEFT HAND BEAM IN THE CYCLE UNDER CONSIDERATION (KIP-IN)
MOMP	MOMENT IN A STORY DUE TO AXIAL LOAD (KIP-IN)
MOMW	TOTAL MOMENT AT FLOOR LEVEL ON THE SHEAR WALL DUE TO END MOMENT AND SHEAR FROM THE RIGHT HAND BEAM (KIP-IN)
MOMWB	MOMENT AT THE RIGHT END OF THE RIGHT HAND BEAM IN THE CYCLE UNDER CONSIDERATION (KIP-IN)
MPC	PLASTIC MOMENT CAPACITY OF COLUMN IN A PARTICULAR STORY (KIP-IN)
MPF	PLASTIC MOMENT CAPACITY OF THE LEFT HAND BEAM (KIP-IN)
MPSW	PLASTIC MOMENT CAPACITY OF THE SHEAR WALL (KIP-IN)
MPW	PLASTIC MOMENT CAPACITY OF THE RIGHT HAND BEAM (KIP-IN)
ND	NUMBER OF DIVISION IN A STORY
NI	NUMBER OF TIMES HORIZONTAL LOAD TO BE INCREMENTED

NM	LENGTH OF A SEGMENT OF A WALL IN A STORY (IN)
NNMM	SEGMENT HEIGHT FROM BASE OF WALL (IN)
NS	NUMBER OF STORIES
ROB	SWAY ROTATION OF RIGHT HAND BEAM
ROFA	A SUBROUTINE SUBPROGRAM
ROS	STORY ROTATION OF FRAME
RMWP	RATIO OF MOMENT TO PLASTIC MOMENT CAPACITY OF WALL
RPWP	RATIO OF CURVATURE TO CURVATURE CORRESPONDING TO RMWP
ROT	ROTATION OF WALL IN A SEGMENT (RAD)
ROTF	ROTATION OF WALL AT THE FLOOR LEVEL (RAD)
ROTFF	JOINT ROTATION OF FRAME (RAD)
ROTO	JOINT ROTATION AT THE BASE OF COLUMN (RAD)
SBF	STIFFNESS OF LEFT HAND BEAM (KIP-IN)
SBW	STIFFNESS OF RIGHT HAND BEAM (KIP-IN)
SC	STIFFNESS OF COLUMN (KIP-IN)
SHEARR	SHEAR AT THE ENDS OF LEFT HAND BEAM (KIP)
SHEARW	SHEAR AT THE ENDS OF RIGHT HAND BEAM (KIP)
SHEC	SHEAR IN COLUMN IN A STORY (KIP)
SR	A SUBROUTINE SUBPROGRAM
SR 1	A SUBROUTINE SUBPROGRAM
SR 3	A SUBROUTINE SUBPROGRAM
SSB	SLOPE OF THE SECOND BRANCH OF SHEAR WALL MOMENT-CURVATURE DIAGRAM
TMOMF	TOP MOMENT IN THE TOP OF THE TOPMOST SEGMENT IN A STORY (KIP-IN)

TMOM	TOP MOMENT IN A SEGMENT OF THE SHEAR WALL (KIP-IN)
VDEF	VERTICAL UPLIFT OF THE BEAM CONNECTED WITH THE WALL (IN)
PPS	TOTAL AXIAL LOAD IN STRUCTURE AT A FLOOR LEVEL (KIP)

APPENDIX A

DERIVATION OF JOINT ROTATION EQUATION

The frame system has been forced into the deformed shape of the wall shown in FIGURE 4.5. Applying the slope-deflection equations, the moment $M_{BW\ i}$, at the end B_i of the right beam is given by

$$M_{BW\ i} = S_{BW\ i} (4 \theta_i + 2 \theta_{WB\ i} - 6 R'_i)$$

where $S_{BW\ i} = EI/L$ is the stiffness of the right beam ($B_i W_i$) and R'_i is the vertical chord rotation of the right beam connecting the column to the wall. Also in the above equation θ_i is the rotation of the frame joint and $\theta_{WB\ i}$ is the rotation at the wall end of the right beam.

Considering the left beam $B_i F_i$ and applying the slope deflection equation, assuming that the joint rotation at the two ends B_i and F_i are equal, the moment, $M_{BF\ i}$, at the end F_i is given by

$$M_{BF\ i} = 6 \cdot S_{BF\ i} \cdot \theta_i$$

where $S_{BF\ i} = EI/L$ is the stiffness of the left hand beam, $B_i F_i$.

The column $B_i B_{i+1}$ between the i th and the $i+1$ th story has a stiffness of $S_{C_{i+1}}$ and the chord rotation of this column is R_{i+1} . The moment, $M_{C_{i+1}}$, at the bottom end B_i of the column $B_i B_{i+1}$ is given by

$$M_{C_{i+1}} = S_{C_{i+1}} (4 \theta_i + 2 \theta_{i+1} - 6 R_{i+1})$$

where θ_{i+1} is the rotation of the frame joint at the $i+1$ th level.

The moment, M_{C_i} , at the top end, B_i , of the column $B_i B_{i-1}$ is given by:

$$M_{C_i} = S_{C_i} (4 \theta_i + 2 \theta_{i-1} - 6 R_i)$$

where $S_{C_i} = EI/L$ is the stiffness of the column $B_i B_{i-1}$ and R_i is the chord rotation of the column. θ_{i-1} is the rotation of the frame joint at the $i-1$ th level.

Equating the sum of the joint moments to zero

$$M_{BW_i} + M_{BF_i} + M_{C_i} + M_{C_{i+1}} = 0$$

Substituting the values of the end moments from the above equations the joint equilibrium equation becomes

$$S_{BW\ i} (4 \theta_i + 2 \theta_{WB\ i} - 6 R'_i) + 6 S_{BF\ i} \theta_i +$$

$$S_{C\ i} (4 \theta_i + 2 \theta_{i-1} - 6 R_i) + S_{C\ i+1}$$

$$(4 \theta_i + 2 \theta_{i+1} - 6 R_{i+1}) = 0$$

and after simplification:

$$\theta_i = \{S_{BW\ i} (6 R'_i - 2 \theta_{WB\ i}) + S_{C\ i} (6 R_i - 2 \theta_{i-1})$$

$$+ S_{C\ i+1} (6 R_{i+1} - 2 \theta_{i+1})\} / \{4 S_{BW\ i} + 4 S_{C\ i} +$$

$$4 S_{C\ i+1} + 6 S_{BF\ i}\} \dots \dots \dots (A-1)$$

Equation (A-1) can be written in the following simplified form

$$\theta_i = \frac{A + B + C + D}{A' + B' + C' + D'} \dots \dots \dots (A-2)$$

Where in the numerator,

$$A = S_{BW\ i} (6 R'_i - 2 \theta_{WB\ i})$$

$$B = S_{C\ i} (6 R_i - 2 \theta_{i-1})$$

$$C = S_{C\ i+1} (6 R_{i+1} - 2 \theta_{i+1})$$

and $D = 0$

And in the denominator,

$$A' = 4 S_{BW\ i}$$

$$B' = 4 S_{C\ i}$$

$$C' = 4 S_{C\ i+1}$$

and $D' = 6 S_{BF\ i}$

Equations (A-1) and (A-2) are valid for the elastic analysis only. When a plastic hinge forms at any one of the potential hinge locations (FIGURE 4.7), the joint rotation equation must be modified. Table A.1 lists the substitutions to be made in Equation (A-2) for the formation of hinges in the various members, so that the joint rotation equation will conform to the particular hinge pattern considered.

In Table A.1, $MP_{BF\ i}$, $MP_{BW\ i}$ are the plastic moment capacities of the beams $B_i F_i$ and $B_i W_i$, and $MP_{C\ i}$ is the plastic moment capacity (reduced for axial load) of the column.

TABLE A.1

MODIFICATION OF ELASTIC SLOPE-DEFLECTION

EQUATIONS FOR JOINT EQUILIBRIUM

Member	Hinge Location	Substitution to be Made In Equation (A-2)
$B_i \quad W_i$		$A = 3 S_{BW \ i} - R'_i - MP_{BW \ i} / 2$
	W_i	$A' = 3 S_{BW \ i}$
	B_i	$A = MP_{BW \ i}$
	OR B_i, W_i	$A' = 0$
$B_i \quad B_{i-1}$	B_{i-1}	$B = 3 S_{C \ i} \cdot R_i - MP_{C \ i} / 2$
		$B' = 3 S_{C \ i}$
	B_i	$B = MP_{C \ i}$
	OR B_i, B_{i-1}	$B' = 0$

Member	Hinge Location	Substitution to be Made In Equation (A-2)
	B_{i+1}	$C = 3 S_{C \ i+1} \cdot R_{i+1} - MP_{C \ i+1} / 2$ $C' = 3 S_{C \ i+1}$
$B_i \ B_{i+1}$	B_i OR B_{i+1}	$C = MP_{C \ i+1}$ $C' = 0$
	F_i	$D = MP_{BF \ i} / 2$ $D' = 3 S_{BF \ i}$
$B_i \ F_i$	B_i OR $B_i \ , \ F_i$	$D = MP_{BF \ i}$ $D' = 0$

APPENDIX B

INCLUSION OF THE P- Δ EFFECT

The P- Δ effect plays a dominant role in the behavior of flexible structures. The vertical load on the structure, P , acting through a sidesway displacement, Δ , produces an additional overturning moment commonly known as the P- Δ moment. Consider a single story structure, which has been displaced by an amount Δ and is subjected to vertical loads, P , as shown in FIGURE B.1. To balance the P- Δ moment an additional shear of $P\Delta/H_s$ is required in the columns of the frame.

In this analysis, the P- Δ effect has been taken into account by analyzing the structure under an equivalent lateral force. In FIGURE B.2a, H_i is the applied lateral load at the i th floor. Due to this applied lateral force system, a lateral deflection Δ_i will be produced.

The story moment between the i th and $i-1$ th floor due to the P- Δ effect is obtained by taking moments about the i th floor of all the axial loads above this level. Similarly, by taking moments at each level, the other story moments are obtained. The extra shear due to the P- Δ effect, V'_i , is given by:

$$V'_i = \frac{P_i (\Delta_i - \Delta_{i-1})}{H_{s_i}}$$

where $H_{s\ i}$ is the story height and P_i is the total vertical load on the structure above the i th floor.

The additional lateral load, $H'_1\ i$, simulating the P- Δ effect can then be determined and is given by

$$H'_1\ i = V'_i - V'_{i+1} = \frac{P_i (\Delta_{1\ i} - \Delta_{1\ i-1})}{H_{s\ i}} - \frac{P_{i+1} (\Delta_{1\ i+1} - \Delta_{1\ i})}{H_{s\ i+1}} \dots \dots \dots (B-1)$$

The structure is now reanalyzed under the lateral force system, $H_i + H'_1\ i$ (without considering vertical loads). In FIGURE B.2b, $\Delta_{2\ i}$ is the lateral deflection corresponding to the new lateral force system. The deflections $\Delta_{2\ i}$ etc. are then compared with the deflections, $\Delta_{1\ i}$ etc. If these deflections do not agree (within a specified convergence limit) the additional horizontal force, $H'_2\ i$, due to the P- Δ effect for the next cycle is given by

$$H'_2\ i = \frac{P_i (\Delta_{2\ i} - \Delta_{2\ i-1})}{H_{s\ i}} - \frac{P_{i+1} (\Delta_{2\ i+1} - \Delta_{2\ i})}{H_{s\ i+1}} \dots (B-2)$$

The structure is again analyzed under the lateral force system $H_i + H'_2\ i$. The process is continued until the changes in lateral deflections are within the convergence limit.

The total vertical load at each floor level has been read into the program. In the present method of simulating the P- Δ effect

a constant vertical load has been used throughout the analysis. In the actual structure the girder shears will introduce tension in one column and compression in the other. This will not change the total vertical load or the gross $P-\Delta$ effect.

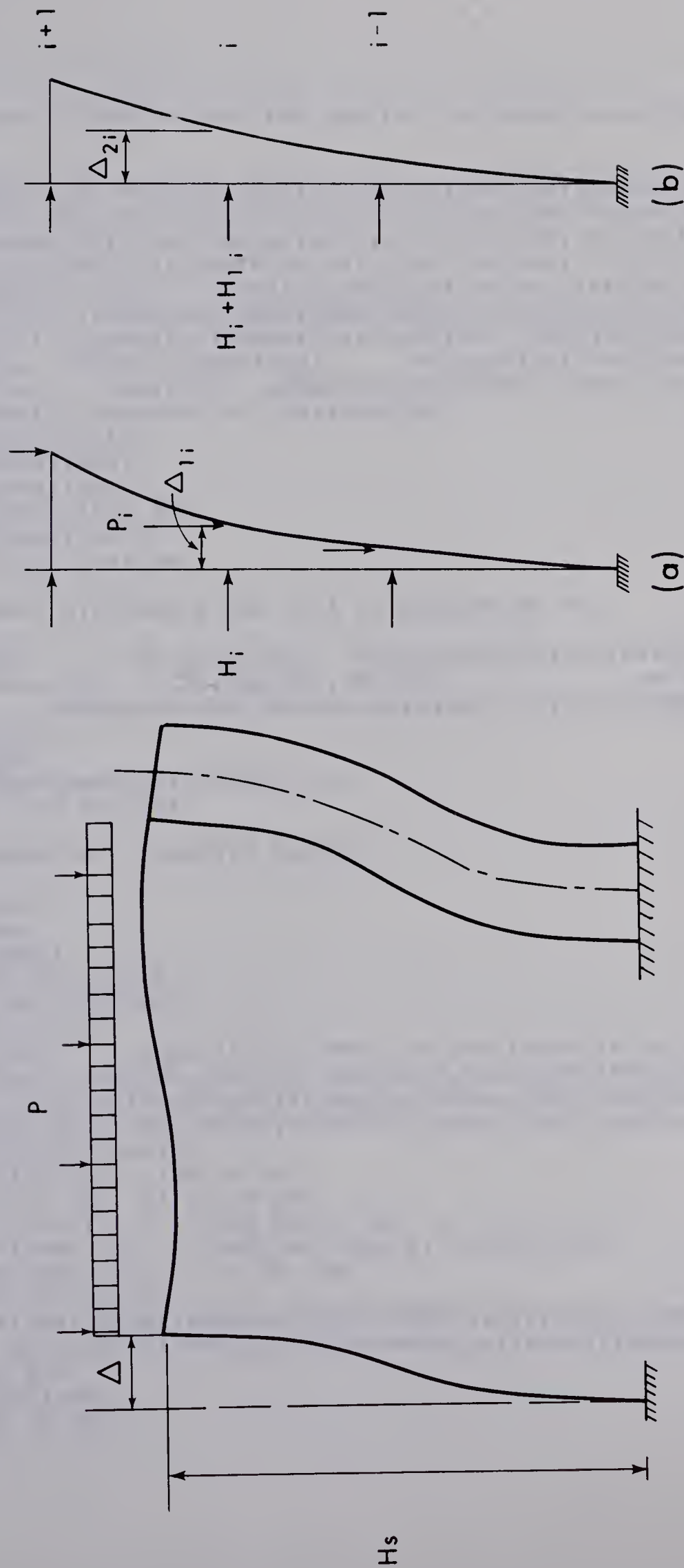


FIGURE B.1 P-Δ EFFECT - SINGLE STORY FRAME

FIGURE B.2 P-Δ EFFECT - MULTI-STORY STRUCTURE


```

C
C      OUTPUT STATEMENTS FOR RESULTS OF FRAME ANALYSIS BY SUBROUTINE ROFA
C      (WITHOUT THE EFFECT OF AXIAL LOADS)
C
235 WRITE(6,240)
240 FORMAT(/45X,'RESULT WITHOUT THE EFFECT OF AXIAL LOAD'//)
    CALL ROFA (ROTFF,SHEC,FOC,MMCB,MMCT,ROTO,NS,NCYCLE,ITER,MMMM)
    DO 242 N=1,NS
        MOMFB(N)=SBF(N)*6.0*ROTFF(ITER,N)
        MOMBF(N)= MOMFB(N)
        SHEARR(N)=2.0*MOMFB(N)/HBF(N)
        WRITE(6,241) N,MOMBW(N),MOMWB(N),MOMBF(N),MOMFB(N),SHEARW(N),SHEAR
1R(N)
241 FORMAT(13X,I3,1X,4F19.2,5X,F10.2,7X,F10.2/)
242 CONTINUE
        WRITE(6,21)
        DO 244 K=1,NS
            F11(K)=F(1,K)-FOC(K)
            F(1,K)=F11(K)
244 CONTINUE
        IX=1
        GO TO 132
245 WRITE(6,22)
C
C      OUTPUT STATEMENTS FOR RESULTS OF SHEAR WALL ANALYSIS BY SUBROUTINE
C      SR3 (WITHOUT THE EFFECT OF AXIAL LOADS)
C
        CALL SR3 (BMOMF,TMOMF,ROTB,NS,F11,FOC,SHEAR,ROTF,DEFF,CROTF,CDEFF,
1MMMM)
        IX=0
        WRITE(6,21)
C
C      ADDITIONAL HORIZONTAL LOAD TO SIMULATE P-DELTA EFFECT
C
251 IF(MMMM .EQ. MAX) GO TO 254
256 PPS(NS+1)=0.0
    HS(NS+1)=5000.0
    CDEFF(MMMM,NS+1)=CDEFF(MMMM,NS)
    F(1,1)=H(NJ,1)+ALPHA*(PPS(1)*CDEFF(MMMM,1)/HS(1)-PPS(2)*(CDEFF(MMM
1M,2)-CDEFF(MMMM,1))/HS(2))
    IF(NS .EQ. 1) GO TO 253
    DO 252 N=2,NS
        F(1,N)=H(NJ,N)+ALPHA*(PPS(N)*(CDEFF(MMMM,N)-CDEFF(MMMM,N-1))/HS(N)
1-PPS(N+1)*(CDEFF(MMMM,N+1)-CDEFF(MMMM,N))/HS(N+1))
252 CONTINUE
253 MMMM=MMMM+1
    GO TO 132
C
C      OUTPUT STATEMENTS FOR RESULTS OF FRAME ANALYSIS BY SUBROUTINE ROFA
C
254 WRITE(6,255)
255 FORMAT(10X,'CONVERGENCE (CALCULATION OF THE EFFECT OF AXIAL LOAD)
1WAS NOT ENOUGH'//)
260 CALL ROFA (ROTFF,SHEC,FOC,MMCB,MMCT,ROTO,NS,NCYCLE,ITER,MMMM)
    DO 270 N=1,NS
        IF(NJ .EQ. 1) GO TO 261
        IF(ABS(DMFB(N)) .GE. MPF(N)) GO TO 262
261 MOMFB(N)=SBF(N)*6.0*ROTFF(ITER,N)
    GO TO 263

```



```

262 MOMFB(N)=DMFB(N)
263 MOMBFB(N)=MOMFB(N)
264 SHEARR(N)=2.0*MOMFB(N)/HBF(N)
    WRITE(6,241)N,MOMBW(N),MOMWB(N),MOMBFB(N),MOMFB(N),SHEARW(N),SHEARR
    1(N)
270 CONTINUE
    DO 271 K=1,NS
    DELTA(NJ,K)=CDEFF(MMMM,K)
    F11(K)=F(1,K)-FOC(K)
    F(1,K)=F11(K)
271 CONTINUE
    IK=1
    GO TO 132
272 WRITE(6,21)

```

```

C
C   OUTPUT STATEMENTS FOR RESULTS OF SHEAR WALL ANALYSIS BY
C   SUBROUTINE SR3
C
    CALL SR3 (BMOMF,TMCMF,ROTB,NS,F11,FOC,SHEAR,ROTF,DEFF,CROTF,CDEFF,
    1MMM)
    WRITE(6,21)

```

```

C
C   DETECTION OF HINGES IN SHEAR WALL.
C

```

```

    IF(NL .NE. JJ) GO TO 273
    IF(ABS(DELTA(NJ,NS)) .GT. STOPD) GO TO 960
273 SSB=(RMWP-1.0)/(RPWP-1.0)
    DO 284 J = 1,L
    D(J)=(TMOM(J)+BMOM(J))/(2.0*MPS(J))
    D1(J)=D(J)*MPS(J)
    IF(ABS(D(J)) .LT. 1.0) GO TO 284
    IF(NL .NE. JJ) GO TO 274
    CPHI(J)=(D(J)-1.0)/SSB + 1.0
    MII(J)=D(J) * MIA(J) / CPHI(J)
274 KP=KP+1
    IF(KP .GE. 2) GO TO 281
275 WRITE(6,280)
280 FORMAT(45X,37HDETECTION OF HINGES IN THE SHEAR WALL///)
281 WRITE(6,282)J,D(J),D1(J)
282 FORMAT(5X,20HHINGE IN SECTION NO.,I3,2X,51H RATIO OF MOMENT IN WALL
    1 TO PLASTIC MOMENT CAPACITY=,F5.2,2X,15HMOMENT IN WALL=,F14.2/)
    WRITE(6,283) MII(J),MIA(J)
283 FORMAT (5X,'REDUCED MOMENT OF INERTIA =',F13.2,3X,'INITIAL MOMENT
    1 OF INERTIA =',F13.2//)
284 CONTINUE
    WRITE(6,21)

```

```

C
C   DETECTION OF HINGES ON FRAME BY SUBROUTINE SR1.
C
    CALL SR1 (DMFB,DMBF,DMBW,DMWB,DMCT,DMCB,MOMFB,MOMBFB,MOMBW,MO
    1MWB,MMCT,MMCB,NS,M3,MPF,MPW,MPC)
    IK=0
    IF(NL .EQ. JJ) GO TO 285
    IF(KP .GE. 1 .OR. M3 .GE. 1) GO TO 295

```

```

C
C   HORIZONTAL LOAD IS INCREMENTED
C

```

```

285 IF(NJ .EQ. NI) GO TO 960
    WRITE(6,21)

```



```

        WRITE(6,291)
291  FORMAT(50X,'HORIZONTAL LOAD INCREMENTED'//)
        WRITE(6,292)
292  FORMAT(10X,'FLOOR NO.',5X,'HORIZONTAL LOAD(K)',5X,'VERTICAL LOAD(
1K)')//)
        DO 294 K=1,NS
            F(1,K)=H(NJ,K)+HLI*FD(K)
            IF(IVL .LE. 0) GO TO 296
            PPS(K)=PPS(K)+HLI*PPD(K)
296  WRITE(6,293)K,F(1,K),PPS(K)
293  FORMAT(13X,I3,11X,F8.2,16X,F8.2//)
294  CONTINUE
        KP=0
        GO TO 302

C
C  HORIZONTAL LOAD IS DECREMENTED
C
C  COMPUTE SECTION HEIGHT FROM BASE IN INCH
C
C
295  IF(NJ .EQ. NI) GO TO 960
        STOPD=30.0*ABS(DELTA(NJ,NS))
        WRITE(6,300)
300  FORMAT(14X,15X,'HORIZONTAL LOAD DECREMENTED TO MAKE THE FRAME ELAS
ITIC SO THAT HORIZONTAL LOAD CAN BE INCREMENTED SLOWLY'//)
        WRITE(6,292)
        DO 301 K=1,NS
            F(1,K)=H(NJ,K)-HLI*FD(K)+HLIR*FD(K)
            IF(IVL .LE. 0) GO TO 303
            PPS(K)=PPS(K)-HLI*PPD(K)+HLIR*PPD(K)
303  DMFB(K)=0.0
        DMBF(K)=0.0
        DMWB(K)=0.0
        DMBW(K)=0.0
        DMCT(K)=0.0
        DMCB(K)=0.0
        WRITE(6,293) K,F(1,K),PPS(K)
301  CONTINUE
        HLI=HLIR
        NL=JJ
        KP=0
302  CONTINUE

C
C  OUTPUT THE ROTATION AND DEFLECTION OF EACH HORIZONTAL LOAD
C
C
960  DO 907 N=1,NS
        WRITE(6,915) N
915  FORMAT(48X,'LOAD-DEFLECTION DATA FOR FLOOR NO. ',I3//)
        WRITE(6,905)
905  FORMAT(10X,'HORIZONTAL LOAD(KIP)',5X,'DEFLECTION(IN)')//)
        DO 906 K = 1,NJ
            WRITE(6,908) H(K,N),DELTA(K,N)
908  FORMAT(16X,F8.2,11X,E13.6//)
906  CONTINUE
907  CONTINUE
400  CONTINUE
        STOP
        END

```


SUBROUTINE SR READS IN DATA - COMPUTES CO-ORDINATES OF EACH FLOOR FROM THE BASE AND THE STIFFNESS OF MEMBERS - ALSO ALL THE INPUT QUANTITIES ARE PRINTED OUT

```

SUBROUTINE SR (L,NS,ND,  EW,KC,KB,FD,NI,HS,DW,PS,      MPF,NNM,
1 MPW,MPC,      SC,SBW,SBF,IVL,PPD,      HBF,HBW,
2      RMWP,RPWP,MAX,CON,HLI,HLIR,HSF,F,MII,MPS,NNMM,MIA,PPS,ALPHA)
  DIMENSION FD(30),HS(30),DW(30 ),PS(30),      SC(30),
1 SBF(30),SBW(30),      HBF(30),
2HBW(30),HSF(30),F(99,30),PSW(30),PPS(30),PPD(30)
  REAL MI(30),MPF(30),MPW(30),MPC(30),MPSW(30),MIC(30),MIBF(30),
1 MIBW(30),NM(30),NNM(300),MII(300),MPS(300),NNMM(300),MIA(300),
2KB,KC
  READ(5,200)KB,KC,EF,EW,NS,ND,MAX,CON,HLI,NI,HLIR
200 FORMAT(1X,2E18.5,2F7.0,3I3,2F5.2,I3,F5.2)
  READ(5,470) RMWP,RPWP,ALPHA,IVL
470 FORMAT(1X,2F7.2,F5.2,I3)
  DO 202 K=1,NS
    READ(5,204)FD(K),HS(K),HBF(K),HBW(K),DW(K),MI(K),MIC(K),
1MIBF(K),MIBW(K)
204 FORMAT(1X,F5.2,4F7.2,4F11.2)
  202 CONTINUE
    DO 102 K=1,NS
      READ(5,101)PS(K) ,PSW(K)
101 FORMAT(1X,2F8.2)
  102 CONTINUE
    DO 205 K = 1,NS
      READ(5,500)      MPF(K),MPW(K),MPC(K),MPSW(K)
500 FORMAT( 17X ,3F14.2,F21.2)
  205 CONTINUE

ALL FORCE UNITS ARE IN KIPS AND ALL LENGTH UNITS ARE
IN INCHES UNLESS STATED OTHERWISE.

L=NS*ND
NDN=0
K=1-ND
CB=0.0
DO 22 J=1,NS
  NM(J)=HS(J)/FLOAT(ND)
  HSF(J)=CB+HS(J)
  CB=HSF(J)
  SBW(J)=EF*MIBW(J)/HBW(J)
  SBF(J)=EF*MIBF(J)/HBF(J)
  SC(J)=EF*MIC(J)/HS(J)
  F(1,J)=FD(J)
  PPS(J)=PS(J)+PSW(J)
  PPD(J)=PPS(J)
  K=K+ND
  NDN=NDN+ND
DO 23 N=K,NDN
  NNM(N)=NM(J)
  MII(N)=MI(J)

```



```

      MIA(N)=MI(J)
      MPS(N) = MPSW(J)
23  CONTINUE
22  CONTINUE
C
C      COMPUTE SECTION HEIGHT FROM BASE IN INCH
C
      BC=0.0
      DO 25 K=1,L
      NNMM(K)=BC+NNM(K)
      BC=NNMM(K)
25  CONTINUE
      WRITE(6,324)
324  FORMAT(1H1)
C
C      OUTPUT STATEMENTS FOR DATA OF FRAME AND SHEAR WALL.
C
      WRITE(6,51)
51  FORMAT(50X,29HDATA FOR SHEAR WALL AND FRAME//)
      WRITE(6,351) NS
351  FORMAT(10X,56HNUMBER OF STORIES =
1    ,I3/)
      WRITE(6,352) ND
352  FORMAT(10X,56HNUMBER OF DIVISION TO BE MADE IN A STORY =
1    ,I3/)
      WRITE(6,355) EW
355  FORMAT(10X,53HMODULUS OF ELASTICITY FOR WALL =
1    ,F9.2/)
      WRITE(6,356) EF
356  FORMAT(10X,53HMODULUS OF ELASTICITY FOR FRAME =
1    ,F9.2/)
      WRITE(6,354) KC
354  FORMAT(10X,43HSPRING CONSTANT AT BASE OF FRAME = ,E19.5/)
      WRITE(6,353) KB
353  FORMAT(10X,43HSPRING CONSTANT AT BASE OF WALL = ,E19.5/)
      WRITE(6,510) CON
510  FORMAT(10X,'CONVERGENCE LIMIT',24X,'=',F20.5/)
      WRITE(6,501) ALPHA
501  FORMAT(10X,'ALPHA',36X,'=',F20.3//)
      WRITE(6,364)
364  FORMAT(1HK)
      WRITE(6,357)
357  FORMAT(10X,10HFLOOR NO./,2X,10HFORCE(KIP),3X,17HMOMENT OF INERTIA,
13X,16HSTORY HEIGHT(IN),3X,14HWALL WIDTH(IN),3X,15HAXIAL LOAD(KIP),
23X,15HAXIAL LOAD(KIP))
      WRITE(6,358)
358  FORMAT(10X,9HSTORY NO.,18X,13HOF WALL (IN4),43X,9HON COLUMN,10X,7H
2ON WALL//)
      DO 10 N=1,NS
      WRITE(6,359)N,FD(N),MI(N),HS(N),DW(N),PS(N),PSW(N)
359  FORMAT(13X,I3,6X,F8.2,7X,F12.2,9X,F8.2,11X,F7.2,9X,F8.2,9X,F8.2/)
10  CONTINUE
      WRITE(6,366)
366  FORMAT(1HK)
      WRITE(6,360)
360  FORMAT(10X,10HFLOOR NO./,2X,14HLENGTH OF BEAM,3X,14HLENGTH OF BEAM
1,3X,17HMOMENT OF INERTIA,3X,17HMOMENT OF INERTIA,3X,17HMOMENT OF I
2NERTIA)
      WRITE(6,361)

```



```

361  FORMAT(10X,9HSTORY NO.,4X,13HON ROLLER(IN),3X,14HCONNECTED WITH,4X
1,14HOF COLUMN(IN4),5X,17HOF BEAM ON ROLLER,3X,17HOF BEAM CONNECTED
2)
WRITE(6,362)
362  FJRMAT(42X,8HWALL(IN),32X,5H(IN4),10X,14HWITH WALL(IN4)//)
DO 71 J=1,NS
WRITE(6,363)J,HB(F(J),HBW(J),MIC(J),MIBF(J),MIBW(J)
363  FORMAT(13X,I3,8X,F8.2,9X,F8.2,9X,F11.2,9X,F11.2,9X,F11.2//)
71  CONTINUE
WRITE(6,411)
411  FORMAT(1HK,30X,'PLASTIC MOMENT CAPACITY OF BEAMS, COLUMNS AND WALL
1'////)
WRITE(6,399)
399  FORMAT(10X,10HFLOOR NO./,2X,
1      15HPLASTIC MOMENT,2X,15HPLASTIC MOMENT,2X,15HPLASTIC MO
3MENT, 5X,15HPLASTIC MOMENT)
WRITE(6,406)
406  FORMAT(10X,9HSTORY NO.,3X,
1      15HCAPACITY OF THE,2X,15HCAPACITY OF THE,2X,15HCAPACITY OF T
2HE, 5X,15HCAPACITY OF THE)
WRITE(6,407)
407  FORMAT(22X,
15HBEAM
2ON ROLLER,2X,15HWALL SIDE BEAM,2X,15HCOLUMN (KIP-IN), 5X,15HWALL
3 (KIP-IN).)
WRITE(6,408)
408  FORMAT(26X,
8H(KIP-IN
1), 7X,8H(KIP-IN)//)
DO 410 K= 1,NS
WRITE(6,409) K,
MPF(K),MPW(K),MPC(K),MPSW(K)
409  FORMAT(13X,I3, 6X,
F14.2,3X,F14.2,3X,F14.2,2X,E18
1.8/)
410  CONTINUE
WRITE(6,385)
385  FORMAT(1HK,45X,39HSTIFFNESSES (EI/L) OF BEAMS AND COLUMNS//)
WRITE(6,365)
365  FORMAT(10X,10HFLOOR NO./,4X,19HSTIFFNESS OF COLUMN,4X,17HSTIFFNESS
1 OF BEAM,4X,17HSTIFFNESS OF BEAM)
WRITE(6,367)
367  FORMAT(10X,9HSTORY NO.,10X,8H(KIP-IN),11X,15HCONNECTED WITH,5X,17
1HON ROLLER(KIP-IN))
WRITE(6,368)
368  FORMAT(49X,12HWALL(KIP-IN)//)
DO 72 J=1,NS
WRITE(6,369)J,SC(J),SBW(J),SBF(J)
369  FORMAT(13X,I3,9X,F15.2,7X,F15.2,6X,F15.2//)
72  CONTINUE
WRITE(6,481)
481  FORMAT(1HK,40X,53HMOMENT CURVATURE RELATIONSHIP OF THE SHEAR
1 WALL//)
WRITE(6,482)
482  FORMAT(10X,9HPOINT NO.,5X,19HRATIO OF MOMENT IN,5X,18HRATIO OF CU
1RVATURE)
WRITE(6,483)
483  FORMAT(24X,19HWALL TO THE PLASTIC,5X,18HIN WALL TO THE)
WRITE(6,484)
484  FORMAT(24X,19HMOMENT CAPACITY OF,5X,18HCURVATURE OF THE)
WRITE(6,485)
485  FORMAT(29X,9HTHE WALL.,10X,18HWALL AT YIELD PT.//)
WRITE(6,488)
RETURN
END

```


SUBROUTINE BAKA COMPUTES THE DISTRIBUTION OF LATERAL LOAD BETWEEN THE FRAME AND SHEAR WALL. MOMENTS AND DEFORMATIONS AT EACH SEGMENT OF THE WALL ARE COMPUTED. THE CONVERGENCE FORMULA IS APPLIED (EXCEPT THE FIRST CYCLE) FOR DEFLECTION AND ROTATION. THE DEFORMATIONS COMPUTED BY THE CONVERGENCE FORMULA ARE ENFORCED ON THE FRAME SYSTEM. THE JOINT ROTATION OF THE FRAME ARE COMPUTED IN SUBROUTINE FRAME.

SUBROUTINE BAKA (IK,IX,MMMM,L,NS,ND,EW,KC,KB,HS,DW,PS,NNM,MPF,MPW,1MPC,SC,SBW,SBF,HBW,MAX,CON,HSF,F,MII,NNMM,TMOM,BMOM,MOMW,ROTB,TMOM2F,BMOMF,ROTF,DEFF,ROTFF,ROTO,ITER,NJ,DMCB,DMCT,DMBF,DMFB,DMBW,3DMWB,MMCB,MMCT,MOMBW,MOMWB,FOC,SHEC,CSHEC,SHEARW,NCYCLE,SHEAR,CDEF4F,CROTF)

DIMENSION BMOM(300),BMOMF(30),CSHEC(30),CDEFF(99,30), CROTF(99,301),DEF(300),DEFF(99,30),DMCB(30),DMCT(30),DMBW(30),DMWB(30),DW(30),2DMFB(30),DMBF(30),F(99,30),FOC(30),HSF(30),HS(30),HBW(30),PS(30),3ROT(300),ROTF(99,30),ROS(30),ROB(30),ROTFF(99,30), ROTO(99),SC(30)4,SBW(30),SBF(30),SHEC(30),SHEARW(30),SHEAR(30),TMOM(300),TMOMF(30)5,VDEF (30)

REAL KB,KC,MOMB,MOMW(30),MII(300),MPC(30),MOMP(30),MPW(30),1MPF(30),MMCB(30),MMCT(30),MOMBW(30),MOMWB(30),NNMM(300),NNM(300)

132 DO 232 M=1,MAX

MOMENTS AT VARIOUS SECTIONS (KIP-IN).

MOMB=0.0

NDN=0

K=1-ND

DO 135 N=1,NS

MOMB=MOMB+F(M,N)*HSF(N)

K=K+ND

NDN=NDN+ND

DO 134 J=K,NDN

RS=0.0

DO 133 I=N,NS

133 RS=RS+F(M,I)*(HSF(I)-NNMM(J))

TMOM(J)=RS

134 CONTINUE

135 CONTINUE

IF(IK .EQ. 1.OR. IX .EQ. 1) GO TO 140

IF(M .EQ. 1) GO TO 145

140 DO 142 N=1,NS

MOMB=MOMB+MOMW(N)

RS=0.0

DO 141 I=N,NS

RS=RS+MOMW(I)

141 CONTINUE

MOMW(N)=RS

142 CONTINUE

NDN=0

K=1-ND

DO 144 I=1,NS

K=K+ND

NDN=NDN+ND

DO 143 J=K,NDN


```

      TMOM(J)=TMOM(J)+MOMW(I)
      BMOM(J)=TMOM(J)+SHEAR(I)*NNM(J)
143  CONTINUE
144  CONTINUE
      GO TO 150
145  BMOM(1)=MOMB
      IF(L .EQ. 1) GO TO 150
      DO 146 J=2,L
146  BMOM(J)=TMOM(J-1)

```

C
C
C

DEFLECTIONS AT VARIOUS SECTIONS (IN)

```

150  A = (TMOM(1)+MOMB)*NNM(1)/(2.0*EW*MII(1))
      ROTB=MOMB/KB
      ROT(1)=ROTB+A
      DEF(1)=ROTB*NNM(1)+A*NNM(1)/2.0
      IF(L .EQ. 1) GO TO 152
      DO 151 J=2,L
      B=(TMOM(J)+BMOM(J))*NNM(J)/(2.0*EW*MII(J))
      ROT(J) = ROT(J-1) + B
      DEF(J)=DEF(J-1)+ROT(J-1)*NNM(J)+B*NNM(J)/2.0
151  CONTINUE

```

C
C
C

MOMENTS, ROTATIONS AND DEFLECTIONS AT EVERY FLOOR LEVEL

```

152  DO 153 N=1,NS
      I=N*ND
      K=I+1-ND
      TMOMF(N)=TMOM(I)
      BMOMF(N)=BMOM(K)
      ROTF(M,N)=ROT(I)
      DEFF(M,N)=DEF(I)
153  CONTINUE
      IF(IX .EQ. 1) RETURN
      IF(IK .EQ. 1) RETURN
      IF(M .EQ. 1) GO TO 155
      DO 154 N=1,NS
      ROTF(M,N)=ROTF(1,N)*ROTF(M-1,N)/(ROTF(M-1,N)-ROTF(M,N))
      DEFF(M,N)=DEFF(1,N)*DEFF(M-1,N)/(DEFF(M-1,N)-DEFF(M,N))
154  CONTINUE
155  ROS(1)=DEFF(M,1)/HS(1)
      IF(NS .EQ. 1) GO TO 161
      DO 160 I=2,NS
      ROS(I)=(DEFF(M,I)-DEFF(M,I-1))/HS(I)
160  CONTINUE
161  DO 162 N=1,NS
      VDEF(N)=-ROTF(M,N)*DW(N)/2.0
      ROB(N)=VDEF(N)/HBW(N)
162  CONTINUE

```

C
C
C

COMPUTE MOMENT, SHEAR AND FORCE ON FRAME BY SUBROUTINE FRAME

```

      CALL FRAME (MAX,ROTF,ROTO,SC,ROS,KC,SBW,SBF,ROB,ROTF,NS,ITER,MMMM
1 , M, DMCB,DMCT,DMBW,DMWB,DMBF,DMFB,MPC,MPF,MPW,NJ,CON)
      IF(NJ .EQ. 1) GO TO 180
      IF(ABS(DMCB(1)) .GE. MPC(1)) GO TO 181
180  MMCB(1)=-KC*ROTO(ITER)
      GO TO 182
181  MMCB(1)=DMCB(1)

```



```

182 IF(NJ .EQ. 1) GO TO 183
    IF(ABS(DMCT(1)) .GE. MPC(1)) GO TO 184
    IF(ABS(DMCB(1)) .GE. MPC(1)) GO TO 185
183 MMCT(1)=SC(1)*(4.0*ROTFF(ITER,1)+2.0*ROTO(ITER)-6.0*ROS(1))
    GO TO 192
184 MMCT(1)=DMCT(1)
    GO TO 192
185 MMCT(1)=SC(1)*(3.0*ROTFF(ITER,1)-3.0*ROS(1))+0.5*DMCB(1)
192 SHEC(1)=(MMCT(1)+MMCB(1))/HS(1)
194 IF(NS .EQ. 1) GO TO 210
    DO 205 K=2,NS
        IF (NJ .EQ. 1) GO TO 195
        IF(ABS(DMCB(K)) .GE. MPC(K)) GO TO 196
        IF(ABS(DMCT(K)) .GE. MPC(K)) GO TO 197
195 MMCB(K)=SC(K)*(4.0*ROTFF(ITER,K-1)+2.0*ROTFF(ITER,K)-6.0*ROS(K))
    GO TO 198
196 MMCB(K)=DMCB(K)
    GO TO 198
197 MMCB(K)=SC(K)*(3.0*ROTFF(ITER,K-1)-3.0*ROS(K))+0.5*DMCT(K)
198 IF(NJ .EQ. 1) GO TO 199
    IF(ABS(DMCT(K)) .GE. MPC(K)) GO TO 200
    IF(ABS(DMCB(K)) .GE. MPC(K)) GO TO 201
199 MMCT(K)=SC(K)*(4.0*ROTFF(ITER,K)+2.0*ROTFF(ITER,K-1)-6.0*ROS(K))
    GO TO 202
200 MMCT(K)=DMCT(K)
    GO TO 202
201 MMCT(K)=SC(K)*(3.0*ROTFF(ITER,K)-3.0*ROS(K))+0.5*DMCB(K)
202 SHEC(K)=(MMCT(K)+MMCB(K))/HS(K)
204 FOC(K-1)=SHEC(K)-SHEC(K-1)
205 CONTINUE
210 FOC(NS)=-SHEC(NS)
    SHEAR(1)=0.0
    DO 219 K=1,NS
        IF(NJ .EQ. 1) GO TO 211
        IF(ABS(DMBW(K)) .GE. MPW(K)) GO TO 212
        IF(ABS(DMWB(K)) .GE. MPW(K)) GO TO 213
211 MOMBW(K)=SBW(K)*(4.0*ROTFF(ITER,K)+2.0*ROTF(M,K)-6.0*ROB(K))
    GO TO 214
212 MOMBW(K)=DMBW(K)
    GO TO 214
213 MOMBW(K)=SBW(K)*(3.0*ROTFF(ITER,K)-3.0*ROB(K))+0.5*DMWB(K)
214 IF(NJ .EQ. 1) GO TO 215
    IF(ABS(DMWB(K)) .GE. MPW(K)) GO TO 216
    IF(ABS(DMBW(K)) .GE. MPW(K)) GO TO 217
215 MOMWB(K)= SBW(K)*(2.0*ROTFF(ITER,K)+4.0*ROTF(M,K)-6.0*ROB(K))
    GO TO 218
216 MOMWB(K)=DMWB(K)
    GO TO 218
217 MOMWB(K)=SBW(K)*(3.0*ROTF(M,K)-3.0*ROB(K))+0.5*DMBW(K)
218 SHEARw(K)=(MOMBW(K)+MOMWB(K))/HBW(K)
    MOMw(K)=-MOMWB(K)-SHEARw(K)*DW(K)/2.0
    F(M+1,K)=-FOC(K)
    SHEAR(1)=SHEAR(1)+F(M+1,K)
219 CONTINUE
    IF(NS .EQ. 1) GO TO 227
    DO 226 K=2,NS
        SHEAR(K)=SHEAR(K-1)-F(M+1,K-1)
226 CONTINUE
227 IF(M .EQ. 1) GO TO 232

```



```

      IF(M .EQ. MAX) GO TO 221
      DO 220 N=1,NS
      IF(ABS((DEFF(M,N)-DEFF(M-1,N))/DEFF(M,N)) .GE. CON .OR. ABS((ROTF(
1M,N)-ROTF(M-1,N))/ROTF(M,N)) .GE. CON) GO TO 232
220 CONTINUE
      GO TO 223
221 WRITE(6,222) MMMM
222 FORMAT(10X,'CYCLE NO.=',I3,7X,'CONVERGENCE (CALCULATION ON WALL) W
1AS NOT ENOUGH'/)
223 NCYCLE=M
      SHEAR(1)=0.0
      DO 224 N=1,NS
      CDEFF(MMMM,N)=DEFF(NCYCLE,N)
      CROTF(MMMM,N)=ROTF(NCYCLE,N)
      IF(MMMM .EQ. 1) CSHEC(N)=SHEC(N)
      SHEAR(1)=SHEAR(1)+F(1,N)
224 CONTINUE
      IF(NS .EQ. 1) GO TO 230
      DO 225 K=2,NS
      SHEAR(K)=SHEAR(K-1)-F(1,K-1)
225 CONTINUE
230 DO 231 K=1,NS
      SHEAR(K)=SHEAR(K)+SHEC(K)
231 CONTINUE
      RETURN
232 CONTINUE
      RETURN
      END

```


IN SUBROUTINE FRAME, THE JOINT ROTATIONS OF THE FRAME FOR A
 SWAYED POSITION (ENFORCED BY THE WALL) ARE COMPUTED.
 THIS IS PERFORMED BY GAUSS-SEIDEL ITERATION METHOD. IF A HINGE
 FORMS IN THE STRUCTURE THE JOINT ROTATION EQUATION IS MODIFIED.

```

SUBROUTINE FRAME (MAX, ROTFF, ROTO, SC, ROS, KC, SBW, SBF, ROB, ROTF, NS, ITR,
  1R, MMMM, M, DMCB, DMCT, DMBW, DMWB, DMBF, DMFB, MPC, MPF, MPW, NJ, CON)
  DIMENSION ROTFF(99,30), ROTO(99), SC(30), ROS(30), SBW(30), SBF(30), ROB
  1(30), ROTF(99,30), DMCB(30), DMCT(30), DMBW(30), DMWB(30), DMBF(30),
  2DMFB(30), AA(30), BB(30), CC(30), DD(30), EE(30), FF(30), GG(30), HH(30), B
  3A(30)
  REAL KC, MPC(30), MPF(30), MPW(30)
  DO 174 I=1, MAX
    IF(I .GT. 1) GO TO 164
    DO 163 K=1, NS
      ROTFF(1,K)=0.0
163 CONTINUE
164 N=I
    IF(N .EQ. 1) GO TO 165
    N=N-1
165 SC(NS+1)=0.0
    ROS(NS+1)=ROS(NS)
    ROTFF(N, NS+1)=ROTFF(N, NS)
    MPC(NS+1)=500.0
    DMCB(NS+1)=0.0
    DMCT(NS+1)=0.0
    IF(NJ .EQ. 1) GO TO 10
    IF(ABS(DMCB(1)) .GE. MPC(1)) GO TO 20
    IF(ABS(DMCT(1)) .GE. MPC(1)) GO TO 11
10  ROTO(I)=(6.0*SC(1)*ROS(1)-2.0*SC(1)*ROTFF(N,1))/(4.0*SC(1)+KC)
    GO TO 20
11  ROTO(I)=(3.0*SC(1)*ROS(1)-0.5*DMCT(1))/(3.0*SC(1)+KC)
20  DO 170 J=1, NS
    IF(NJ .EQ. 1) GO TO 21
    IF(ABS(DMBW(J)) .GE. MPW(J)) GO TO 30
    IF(ABS(DMWB(J)) .GE. MPW(J)) GO TO 40
21  AA(J)=SBW(J)*(6.0*ROB(J)-2.0*ROTF(M,J))
    EE(J)=4.0*SBW(J)
    GO TO 50
30  AA(J)=-DMBW(J)
    EE(J)=0.0
    GO TO 50
40  AA(J)=3.0*SBW(J)*ROB(J)-0.5*DMWB(J)
    EE(J)=3.0*SBW(J)
50  IF(NJ .EQ. 1) GO TO 51
    IF(ABS(DMBF(J)) .GE. MPF(J)) GO TO 60
    IF(ABS(DMFB(J)) .GE. MPF(J)) GO TO 70
51  BB(J)=0.0
    FF(J)=6.0*SBF(J)
    GO TO 80
60  BB(J)=-DMBF(J)
    FF(J)=0.0
    GO TO 80
70  BB(J)=-0.5*DMFB(J)
    FF(J)=3.0*SBF(J)

```



```

80 IF(NJ .EQ. 1) GO TO 81
   IF (ABS(DMCT(J)) .GE. MPC(J)) GO TO 90
   IF (ABS(DMCB(J)) .GE. MPC(J)) GO TO 100
81 IF(J .EQ. 1) GO TO 82
   CC(J)=SC(J)*(6.0*ROS(J)-2.0*ROTF(I,J-1))
   GO TO 83
82 CC(J)=SC(J)*(6.0*ROS(J)-2.0*ROTD(I))
83 GG(J)=4.0*SC(J)
   GO TO 110
90 CC(J)=-DMCT(J)
   GG(J)=0.0
   GO TO 110
100 CC(J)=3.0*SC(J)*ROS(J)-0.5*DMCB(J)
   GG(J)=3.0*SC(J)
110 IF(NJ .EQ. 1) GO TO 111
   IF(ABS(DMCB(J+1)) .GE. MPC(J+1)) GO TO 120
   IF(ABS(DMCT(J+1)) .GE. MPC(J+1)) GO TO 130
111 DD(J)=SC(J+1)*(6.0*ROS(J+1)-2.0*ROTF(N,J+1))
   HH(J)=4.0*SC(J+1)
   GO TO 140
120 DD(J)=-DMCB(J+1)
   HH(J)=0.0
   GO TO 140
130 DD(J)=3.0*SC(J+1)*ROS(J+1)-0.5*DMCT(J+1)
   HH(J)=3.0*SC(J+1)
140 BA(J)=EE(J)+FF(J)+GG(J)+HH(J)
   IF(ABS(BA(J)) .LT. 0.0001) GO TO 170
   ROTFF(I,J)=(AA(J)+BB(J)+CC(J)+DD(J))/BA(J)
170 CONTINUE
171 ITER=I
   IF(ITER .EQ. 1) GO TO 174
   IF(ABS(DMCB(1)) .GE. MPC(1)) GO TO 176
   IF(ABS((ROTD(I)-ROTD(I-1))/ROTD(I)) .GT. CON ) GO TO 173
176 DO 172 K=1,NS
   IF(ABS(BA(K)) .LE. 0.0001) GO TO 172
   IF(ABS((ROTF(I,K)-ROTF(I-1,K))/ROTF(I,K)) .GT. CON ) GO TO 173
172 CONTINUE
   RETURN
173 IF(I .EQ. MAX) GO TO 175
174 CONTINUE
175 WRITE(6,181) M,MMMM
181 FORMAT(10X,'CYCLE NO.=' ,I3,2X,'TO' ,I3,7X,'CONVERGENCE (CALCULATION
1 ON FRAME) WAS NOT ENOUGH'/)
   RETURN
   END

```


OUTPUT STATEMENTS FOR FRAME FORCES AND DEFORMATIONS

```

SUBROUTINE ROFA (ROTFF, SHEC, FOC, DMCB, DMCT, ROTO, NS, NCYCLE, ITER, MMM
1)

```

```
DIMENSION ROTFF(99,30),SHEC(30),FOC(30),ROTO(30),DMCB(30),DMCT(30)
WRITE(6,371)
```

```
371  FORMAT(50X,28HRESULTS  OF  FRAME  ANALYSIS///)
```

```
WRITE(6,372) MMMM,NCYCLE
```

```
372  FORMAT(10X, 'CYCLE NO.   MMMM=', I3, 2X, 'NCYCLE=', I3///)
```

WRITE(6,200)

200 FORMAT(10X,7HCOLUMNS//)

WRITE(6,373)

```

373  FORMAT(10X,10HFLOOR NO./,3X,19HJOINT ROTATION(RAD),3X,18HBOTTOM MO
      1MENT(KIN),3X,16HTOP  MOMENT(KIN),3X,10HSHEAR(KIP),3X,10HFORCE(KIP)

```

21

WRITE(6,374)

```
374  FORMAT(10X,9HISTORY NO.//)
```

```
WRITE(6,375)ROTC(ITER)
```

```
375  FORMAT(12X,4HBASE,10X,E13.6/)
```

DO 377 K = 1, NS

WRITE(6,378)K,ROTF(ITER,K),DMCB(K),DMCT(K),SHEC(K),FOC(K)

```
378  FORMAT(13X,I3,10X,E13.6,8X,F14.2,6X,F14.2,4X,F10.2,3X,F10.2/)
```

377 CONTINUE

```
814 WRITE(6,386)
```

386 FORMAT(1HK)

WRITE(6,201)

```
201 FORMAT(//10X,5HBEAMS//)
```

WRITE(6,380)

380 FORMAT(10X,9HFLOOR NO.,3X,16HMOMENT IN WALL,3X,16HMOMENT IN WA
1LL,3X,16HMOMENT AT COLUMN,3X,16HMOMENT AT ROLLER,3X,14HSHEAR AT
2THE,3X,14HSHEAR AT THE)

WRITE(6,381)

```

381  FORMAT(22X,16H SIDE BEAM AT THE,3X,16H SIDE BEAM AT THE,3X,16H END OF
      1  THE BEAM,3X,16H END OF  THE BEAM,3X,14H ENDS  OF  WALL,3X,14H ENDS
      2  OF  BEAM)

```

WRITE(6,382)

```
382  FORMAT(22X,16HCOLUMN END(K-IN),3X,16HWALL  END (K-IN),3X,16HON  RO  
1LLER(K-IN),3X,16HON  ROLLER(K-IN),3X,14HSIDE BEAM(KIP),3X,14HON  RO  
2LLER(KIP))//)
```

RETURN

END

C
C
C
C
C

OUTPUT STATEMENTS FOR SHEAR WALL FORCES AND DEFORMATIONS

```

SUBROUTINE SR3 (BMOMF,TMOMF,ROTB,NS,F11,FOC,SHEAR,ROTF,DEFF,CROTF,
1CDEFF,MMMM)
  DIMENSION F11(30),FOC(30),SHEAR(30),ROTF(99,30),DEFF(99,30),
1CROTF(99,30),CDEFF(99,30),BMOMF(30),TMOMF(30)
  WRITE(6,55)
55  FORMAT(39X,52HSHEAR WALL ANALYSIS AND FINAL SLOPES AND DEFLECTIONS
1//)
  WRITE(6,312)
312  FORMAT(3X,10HFLOOR NO./,3X,15HWALL FORCE(KIP),3X,16HFRAME FORCE(KI
1P),3X,15HWALL SHEAR(KIP),9X,17HWALL MOMENT(K-IN),9X,10HSLOPE(RAD),
23X,14HDEFLECTION(IN))
  WRITE(6,401)
401  FORMAT(3X,9HSTORY NO.,62X,6HBOTTOM,13X,3HTOP//)
  WRITE(6,20)ROTB
  20  FORMAT(5X,4HBASE,93X,E13.6/)
  DO 400 K=1,NS
  WRITE(6,313) K,F11(K),FOC(K),SHEAR(K),BMOMF(K),TMOMF(K),ROTF(1,K),
1DEFF(1,K)
313  FORMAT(6X,13,7X,F8.2,11X,F8.2,9X,F10.2,5X,F14.2,3X,F14.2,4X,E13.6,
13X,E13.6/)
400  CONTINUE
  WRITE(6,404)
404  FORMAT(1HK,50X,31HCHECK ON SLOPES AND DEFLECTIONS//)
  WRITE(6,405)
405  FORMAT(5X,9HFLOOR NO.,5X,10HSLOPE(RAD),4X,14HDEFLECTION(IN)//)
  DO 402 N=1,NS
  WRITE(6,403) N,CROTF(MMMM,N),CDEFF(MMMM,N)
403  FORMAT(8X,13,7X,E13.6,4X,E13.6/)
402  CONTINUE
  RETURN
  END

```

OF COMPILATION *****

IN SUBROUTINE SR1 FORMATION OF PLASTIC HINGE IN A MEMBER OF
THE FRAME IS DETECTED

```

SUBROUTINE SR1 (DMFB,DMBF,DMBW,DMWB,DMCT,DMCB,MOMFB,MOMBF,MOMBW,MO
1MWB,MMCT,MMCB,NS,M3,MPF,MPW,MPC)
  DIMENSION DMFB(30),DMBF(30),DMBW(30),DMWB(30),DMCT(30),DMCB(30)
  REAL      MOMFB(30),MOMBF(30),MOMBW(30),MOMWB(30),MMCT(30),
1MMCB(30),MPF(30),MPW(30),MPC(30)

```

```

  M3=0

```

```

  DO 403 K=1,NS

```

```

    DMFB(K)=MOMFB(K)

```

```

    DMBF(K)=MOMBF(K)

```

```

    DMBW(K)=MOMBW(K)

```

```

    DMWB(K)=MOMWB(K)

```

```

    DMCT(K)=MMCT(K)

```

```

    DMCB(K)=MMCB(K)

```

```

403  CONTINUE

```

```

  DO 610 K=1,NS

```

```

    WRITE(6,633)

```

```

633  FORMAT(1HK)

```

```

    IF((ABS(DMCB(K))).LT.MPC(K))GO TO 616

```

```

    M3=M3+1

```

```

    WRITE(6,617)K,DMCB(K)

```

```

617  FORMAT(10X,50HHINGE AT BOTTOM POINT OF COLUMN IN STORY NO.,I

```

```

13,5X,40HMOMENT AT BOTTOM POINT OF COLUMN =,F14.2,5H K-IN/)

```

```

    IF(DMCB(K) .LT. 0.0) GO TO 10

```

```

    DMCB(K)=MPC(K)+0.0001

```

```

    GO TO 616

```

```

10  DMCB(K)=-MPC(K)-0.0001

```

```

616  IF((ABS(DMCT(K))).LT.MPC(K))GO TO 602

```

```

    M3=M3+1

```

```

    WRITE(6,618)K,DMCT(K)

```

```

618  FORMAT(10X,50HHINGE AT TOP POINT OF COLUMN IN STORY NO.,I

```

```

13,5X,40HMOMENT AT TOP POINT OF COLUMN =,F14.2,5H K-IN/)

```

```

    IF(DMCT(K) .LT. 0.0) GO TO 20

```

```

    DMCT(K)=MPC(K)+0.0001

```

```

    GO TO 602

```

```

20  DMCT(K)=-MPC(K)-0.0001

```

```

602  IF((ABS(DMFB(K))).LT.MPF(K))GO TO 604

```

```

    M3=M3+1

```

```

    WRITE(6,603)K,DMFB(K)

```

```

603  FORMAT(10X,50HHINGE AT ROLLER END OF BEAM ON ROLLER OF FLOOR NO.,I

```

```

13,5X,40HMOMENT AT ROLLER END OF BEAM ON ROLLER =,F14.2,5H K-IN/)

```

```

    IF(DMFB(K) .LT. 0.0) GO TO 30

```

```

    DMFB(K)=MPF(K)+0.0001

```

```

    GO TO 604

```

```

30  DMFB(K)=-MPF(K)-0.0001

```

```

604  IF((ABS(DMBF(K))).LT.MPF(K))GO TO 606

```

```

    M3=M3+1

```

```

    WRITE(6,605)K,DMBF(K)

```

```

605  FORMAT(10X,50HHINGE AT COLUMN END OF BEAM ON ROLLER OF FLOOR NO.,I

```

```

13,5X,40HMOMENT AT COLUMN END OF BEAM ON ROLLER =,F14.2,5H K-IN/)

```

```

    IF(DMBF(K) .LT. 0.0) GO TO 40

```



```
DMBF(K)=MPF(K)+0.0001
GO TO 606
40 DMBF(K)=-MPF(K)-0.0001
606 IF((ABS(DMBW(K))).LT.MPW(K))GO TO 608
M3=M3+1
WRITE(6,607)K,DMBW(K)
607 FORMAT(10X,50HHINGE AT COLUMN END OF WALL SIDE BEAM OF FLOOR NO.,I
13,5X,40HMOMENT AT COLUMN END OF WALL SIDE BEAM =,F14.2,5H K-IN/)
IF(DMBW(K) .LT. 0.0) GO TO 50
DMBW(K)=MPW(K)+0.0001
GO TO 608
50 DMBW(K)=-MPW(K)-0.0001
608 IF((ABS(DMWB(K))).LT.MPW(K))GO TO 610
M3=M3+1
WRITE(6,609)K,DMWB(K)
609 FORMAT(10X,50HHINGE AT WALL END OF WALL SIDE BEAM OF FLOOR NO.,I
13,5X,40HMOMENT AT WALL END OF WALL SIDE BEAM =,F14.2,5H K-IN/)
IF(DMWB(K) .LT. 0.0) GO TO 60
DMWB(K)=MPW(K)+0.0001
GO TO 610
60 DMWB(K)=-MPW(K)-0.0001
610 CONTINUE
RETURN
END
```


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